CLASS NOTES OF
"ENGG. MECHANICS"

Technical Classes

✅ Technical Classes के Course में उपलब्ध फीचर्स

1. सभी कक्षाएं स्मार्ट बोर्ड पर Live
2. विशेषज्ञ शिक्षको द्वारा पठन पाठन
3. Lecture के दौरान डाउट सॉल्विंग
4. Recorded Lecture (VOD)
5. प्रत्येक क्लास का PDF नोट्स
6. SBTE Exam की तैयारी के लिए ब्रह्मास्त्र क्लास
7. Laptop में क्लास देखने की सुविधा

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नोट :-

1. सभी ऑफलाइन कक्षाएं Technical Classes के कैंपस तथा सभी ऑनलाइन कक्षाएं Technical Classes के एप्लीकेशन पर चलेगी।

2. यह नोट्स टेक्निकल क्लासेस के स्टूडेंट्स के लिए है, तथा क्लास करने के बाद अधिक प्रभावी होगा।

Nitesh Sir

I) Course Curriculum Detailing: This course curriculum detailing depicts learning outcomes at course level and session level and their attainment by the students through Classroom Instruction (CI), Laboratory Instruction (LI), Term Work (TW) and Self Learning (SL). Students are expected to demonstrate the attainment of Theory Session Outcomes (TSOs) and Lab Session Outcomes (LSOs) leading to attainment of Course Outcomes (COs) upon the completion of the course. While curriculum detailing, NEP 2020 related reforms like Green skills, Sustainability, Multidisciplinary aspects, Society connect, Indian Knowledge System (IKS) and others must be integrated appropriately.

J) Theory Session Outcomes (TSOs) and Units: T2425104

Major Theory Session Outcomes (TSOs)	Units	Relevant COs Number(s)
TSO 1a. Explain concepts of the given terms. *TSO 1b.* Use relevant units of various quantities in the given situations. *TSO 1c.* Explain effects of a force on the given object. *TSO 1d.* Resolve the given single force. *TSO 1e.* Calculate the resultant of the given force system. *TSO 1f.* Find the resultant of the given force system using law of parallelogram *TSO 1g.* Determine graphically the resultant of the given force system by triangle law and polygon law.	**Unit-1.0 Mechanics and Force System** 1.1 Significance and relevance: Mechanics, applied mechanics, statics and dynamics. 1.2 Space, time, mass, particle, body, rigid body. 1.3 Scalar and vector quantity, Units of measurement (SI units) Fundamental units and derived units. 1.4 Force - unit, representation as a vector and by Bow's notation, characteristics and effects of a force, Principle of transmissibility of force. Force system and its classification. 1.5 Resolution of a force - Orthogonal and Non-Orthogonal components of a force, moment of a force, Avignon's Theorem. 1.6 Composition of forces - Resultant, analytical method of determination of resultant for concurrent, non-concurrent and parallel co-planar force systems -Law of triangle, Law of parallelogram and law of polygon of forces. 1.7 Graphic statics, graphical representation of force, Space diagram, force diagram, polar diagram and funicular polygon, Graphical method of determination of resultant for concurrent and parallel co-planar force systems.	**CO1, CO2**
TSO 2a. Draw the free body diagram for the given condition. *TSO 2b.* Determine unknown force in the given situation using Lami's theorem. *TSO 2c.* Identify the types of beams required for the given situation. *TSO 2d.* Determine reactions in the given type of beam analytically. *TSO 2e.* Solve problems using free body diagram and Lami"s theorem.	**Unit-2.0 Static Equilibrium** 2.1 Equilibrium and Equilibrant, Free body and Free body diagram, Analytical and graphical conditions of equilibrium. 2.2 Equilibrium of force systems analytically 2.3 Lami's Theorem. 2.4 Types of beam (determinate and indeterminate), supports (simple, hinged, roller and fixed) and loads acting on beam (vertical and inclined point load, distributed load, load, couple), span of beam. 2.5 Beam reaction for cantilever, simply supported beam with or without overhang - subjected to combination of Point load and LTD load or Vertical Point load and couple. 2.6 Beam reaction for simply supported beam subjected to vertical loads only.	**CO1, CO2**

Major Theory Session Outcomes (TSOs)	Units	Relevant COs Number(s)
TSO 3a. Calculate force of friction and coefficient of friction for the given condition or situation *TSO 3b.* Describe the conditions for friction for the given situation. *TSO 3c.* Identify the various forces acting on a ladder for the given conditions using free body diagram. *TSO 3d.* Compare the value of coefficient of friction between different surfaces. *TSO 3e.* Interpret the effect of change of masses, change of angle of inclination or both on the coefficient of friction *TSO 3f.* Calculate forces acting on a body that is moving on a horizontal rough surface *TSO 3g.* Determine the forces acting on a body that is moving on an inclined plane	**Unit 3.0 Friction** 3.1 Friction and its relevance in engineering, types and laws of friction, limiting equilibrium, limiting friction, co-efficient of friction, angle of friction, angle of repose, relation between co-efficient of friction and angle of friction. 3.2 Equilibrium of bodies on level surface subjected to force parallel and 3.3 inclined to plane. 3.4 Equilibrium of bodies on inclined plane subjected to force parallel to the plane only. FBD of ladder in friction	**CO3, CO4**
TSO 4a. Distinguish between centroid and center of gravity *TSO 4b.* Calculate the centroid of geometrical plane figures. *TSO 4c.* Calculate centroid of the given composite plane lamina *TSO 4d.* Determine centre of gravity of the given simple solid. *TSO 4e.* Determine centre of gravity of the given composite solid. *TSO 4f.* Calculate Moment of Inertia of different geometric shapes.	**Unit 4.0 Centroid, Centre of Gravity and Moment of Inertia** 4.1 Introduction to Centroid, Centre of Gravity and Areas 4.2 Centroid of geometrical plane figures (square, rectangle, triangle, circle, semi-circle, quarter circle). 4.3 Centroid of composite figures composed of not more than three geometrical figures and centroid of perforated section, axis of symmetry 4.4 Centre of Gravity of simple solids (Cube, cuboid, cone, cylinder, sphere, hemisphere). 4.5 Centre of Gravity of composite solids composed of not more than two simple solids. 4.6 Moment of inertia - Introduction, calculation of moment of inertia by integration method, theorem of perpendicular axis, theorem of parallel axis, moment of inertia of a rectangular section, hollow rectangular section, circular section, hollow circular section, triangular section	**CO4**
TSO.5a Describe the components of the given lifting machine. TSO.5b Differentiate the working principle of the given two types of lifting machines. TSO.5c Determine velocity ratio, efficiency of the given lifting machine. TSO.5d Calculate effort required and load lifted by the given lifting machine. TSO.5e Draw the graph with the given data TSO.5f Interpret the given graphs TSO.5g Select the relevant lifting machine for the given purpose with justification	**Unit-5.0 Simple Lifting Machine** 5.1 Simple lifting machine, load, effort, mechanical advantage, Applications and advantages. Velocity ratio, efficiency of machines, Law of machine. 5.2 Ideal machine, friction in machine, maximum Mechanical advantage and efficiency, reversible and non-reversible machines, condition for reversibility 5.3 Velocity ratios of Simple axle and wheel, Differential axle and wheel, Worm and worm wheel, Single purchase and double purchase crab winch, Screw jack, Weston's differential pulley block, geared pulley block.	**CO2, CO5**

Major Theory Session Outcomes (TSOs)	Units	Relevant COs Number(s)
	5.4 Graphs of Load verses Effort, Load verses ideal Effort, Load verses Effort lost in friction, Load verses MA, Load verses Efficiency.	

Note: One major TSO may require more than one Theory session/Period.

K) Suggested Laboratory (Practical) Session Outcomes (LSOs) and List of Practical: P2425104

Practical/Lab Session Outcomes (LSOs)	S. No.	Laboratory Experiment/Practical Titles	Relevant COs Number (s)
LSO 1.1. Use force polygon table to determine the resultant of concurrent forces	1.	Determine resultant of concurrent coplanar force system using force polygon table.	CO1, CO2
LSOs 2.1 Apply Lami's theorem LSOs 2.2 Use simply supported beams to find reactions	2.	Determine unknown force in a concurrent balance force system using Lami's Theorem.	CO1, CO2
	3	Find reactions at the supports of a simply supported beam and compare the results with analytical values.	
	4	Determine the support reactions for simply supported beam by • Beam reaction apparatus • Circular dial type weight	
LSO 3.1. Apply law of friction on horizontal plane and inclined plane *LSO 3.2.* Coefficient of friction between different materials *LSO 3.3.* Coefficient of friction between belt and pulley.	5	Determine coefficient of friction on horizontal and inclined plane.	CO2, CO3
	6	Determine the co efficient of friction between two surfaces by • angle of repose methods • friction plane method	
	7	Find the coefficient of friction between belt and pulley in a belt friction set up.	
LSO 4.1. Determine the centroid of different geometrical figures.	8	Determine the centroid of geometrical plane figures (squares, rectangle, triangle)	CO4
LSO 4.2. Find moment of inertia	9	Determine the moment of inertia of a fly wheel	
LSOs 5.1 Use simple screw jack *LSOs 5.2* Use differential axle and wheel *LSOs 5.3* Use single and double purchase crab winch *LSOs 5.4* Use jib crane *LSOs 5.5* Use worm and worm wheel apparatus	10	Find M.A, V.R and efficiency of screw jack.	CO5
	11	Find M.A, V.R and efficiency of differential wheel and axle	
	12	Calculate the efficiency of single purchase crab winch and double purchase crab winch	
	13	Determine forces in jib crane.	
	14	Determine the efficiency of worm and worm wheel.	

Engineering Mechanics.

→ Numerical + conceptual subject.

→ How to cover this cover/study :—

Step-I prepare a main Notes.

Step-II Memorise the daily class notes. (At Home)

" III write down 2 times in rough copy.

Step IV # Short-Notes :—

→ formulae (unit-wise)

→ Some importants points.

Step V Solve previous Semester question

Step VI Book :- (I) Engineering Mechanics by R.S Khurmi

(II) Mechanics of Solid by Beer & Johnston.

UNIT-01.

Basics of Mechanics and Force System.

2 Question object

Marks – 2×2 → 4 Marks.

2 → Subjective Question.

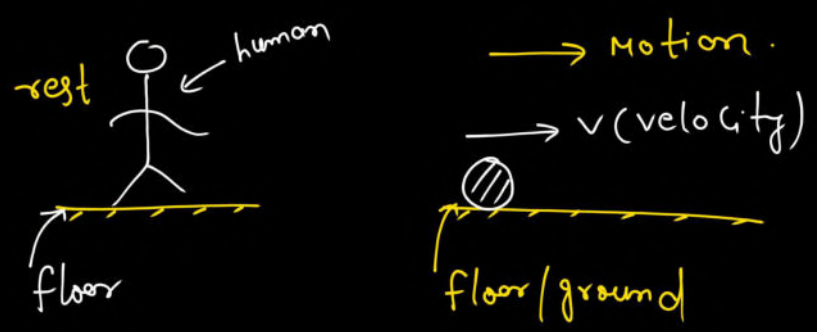
Marks :— 2×4 = 8 Marks ✓

1×4 + 1×6 = 10 Marks ✓

1×6 + 1×6 = 12 Marks

MECHANICS :→

→ Mechanics is the branch of natural physics which deals with the state of rest or motion of an object/body under action of external system of force.



Engineering Mechanics :→

→ Engineering mechanics is deals with mechanics (Rest or motion) of rigid body/object.

Rigid body :→

→ When shape and size of an object remains constant upto application of certain amount external deforming force, then object is called Rigid object or rigid body.

OR

→ When deformation in an object is negligible under application of certain amount external deforming then object is called rigid object

OR

→ When distance between two point on an object remains constant then this object is called rigid body.

NOTE :→

(a) **Deformation :→** Change in shape and size of an object is called deformation

(b) No body/object in the universe is perfectly or absolutely rigid body that is all body or objects are deformable or flexible or non rigid body.

Classification of Engg. Mechanics :→

→ There are two branches of Mechanics

I Statics.

II Dynamics.

I Statics :→

→ It is the branch of mechanics which deals with state of rest of an object.

II Dynamics :→

→ It is the branch of mechanics which deals with the State of motion of an object.

→ Further Dynamics is divided in to two parts :—

ⓐ Kinematics

ⓑ Kinetics.

ⓐ Kinematics :→

→ It is the part of dynamics which deal or in which we study about motion (velocity, acceleration, displacement) of the object without knowing cause behind motion.

$$\boxed{\begin{aligned} S &= ut + \frac{1}{2}at^2 \\ V &= u + at \\ V^2 &= u^2 + 2aS \end{aligned}}$$

⇓

Equation of Kinematics.

II Kinetics :—

→ It is a part of dynamics in which we study about cause/reason (force, moment) behind motion.

→ Equation of Kinetics or equation of motion :—

Newton's 2nd law :— $\boxed{\vec{F}_{net} = m\vec{a}}$

Mechanics

- Statics (Rest)
- Dynamics (Motion)
  - Kinematics (velocity, acceleration, displacement)
  - Kinetics (force, moment)

Engg. Mechanics.

Physical quantity :→

→ Those quantities, which can be measured are called physical quantities.

example: mass, length, Time, speed, velocity, distance, acceleration, force, current, Area, volume, pressure, density, momentum etc.

→ All kind of emotions that can't be measured are not physical quantities,

example, Love, ~~Hate~~, Ego, brilliance, etc

These are not physical quantit

→ **Measurement:→**

→ The process of comparing with a standard value is called measurement

→ To represent a measured value of a physical quantity, we have two thing, a numerical value and a unit of that physical quantity.

example:→

→ Measurement of mass = $\underbrace{20}_{\text{Numerical value}}$ $\overset{\text{unit of mass.}}{\text{Kg}}$

→ Measure value of 'distance' = $\underbrace{100}_{\text{Numerical value}}$ $\underbrace{\text{meter}}_{\text{unit of distance}}$

→ Measured value of 'Time' = 30 min.

Numerical value

Unit of time

UNIT :→ Unit is a standard value of a physical quantity, which is use to measure that physical quantity.

Fundamental Physical quantities :→

→ Those physical quantities which are Independant from any other physical quantities are called Fundamental physical quantities.

** → Fundamental physical quantities also called as Basic physical quantities.

example :-

Fundamental quantities :→ Mass, length, Time etc.

Derived Physical quantities :→

$$\text{Speed} = \frac{\text{Distance}}{\text{time}}$$

Distance → Basic physical quantity

time → Basic physical quantity.

Speed → Derived physical quantity.

* velocity = $\frac{\text{Displacement}}{\text{Time}}$

Displacement → Basic P.Q

Time → Basic P.Q

velocity → Derived Physical quantity

* acceleration(a) = $\frac{\text{Change in velocity}}{\text{Time}}$

acceleration → Derived physical quantity.

→ Those physical quantities which combination of two or more than two Basic or fundamental physical quantities are called derived physical quantities.

example :- speed, velocity, acceleration, Force, momentum, density, pressure etc.

Engg. Mechanics

Fundamental unit :→

→ units of fundamental physical quantities are called fundamental units.

example: MASS → Unit Kg → Fundamental Unit.

SI system of unit :→

→ In this system there are '7' fundamental physical quantities and '2' supplementry physical quantities.

	Fundamental physical quantities	Fundamental SI unit
I.	Mass →	Kilogram (Kg)
II.	length →	meter (m)
III.	Time →	Second (Sec.)
IV.	Temperature →	Kelvin (K)
V.	Current →	ampere (A)
VI.	luminous intensity →	Candela (Cd)
VII.	Amount of Substance →	Mole (mol.)

⇔ Two supplementry physical quantities :—

	Supplementry physical quantity	SI unit
I	Plane Angle →	Radian (rad.)
II	Solid Angle. →	Steradian (Sr.)

Derived unit :→ $x^{-1} = \frac{1}{x}$

→ unit of a derived physical quantities is called derived unit.

example :- Speed = $\frac{\text{length}}{\text{Time}}$ → unit $\frac{\text{meter}}{\text{Second}}$ derived unit.

Speed → derived physical quantity

$= \frac{m}{Sec.}$

$= m/Sec$

$= m\,Sec.^{-1}$

example :-

Acceleration (a) = $\frac{v - u}{t}$ → unit $\frac{\frac{\text{meter}}{\text{Sec.}}}{\text{Sec.}}$

v → final velocity, u → initial velocity, t → time

Acceleration (a) → Derived Physical quantity

$= \frac{\text{meter}}{Sec^2}$

$= m/Sec^2$

$= m\,Sec^{-2}$

Derived unit.

example :-

Force = mass × Acceleration.

Force → Derived physical quantity.

SI unit

$Kg\,\frac{m}{Sec^2}$ = newton (N) — Derived unit

Engg. Mechanics.

Some important prefixes used before unit :→

I prefixes which made unit bigger :-

prefixes	value
(a) Deca (da) ←→	10^1
(b) Hecto (ha) ←→	$10^2 = 100$
(c) Kilo (K) ←→	$10^3 = 1000$
(d) Mega (M) ←→	$10^6 = 1000000$
(e) Giga (G) ←→	$10^9 =$
(f) Tera (T) ←→	10^{12}
(g) Penta (P) ←→	10^{15}

example :- meter → SI unit

→ Kilo meter → 10^3 meter

example Sec. → SI unit.

→ Kilo Sec ⟷ 10^3 sec.

→ M Sec ⟷ 10^6 sec.

→ 10^9 sec ⟷ G Sec

example $\frac{\text{meter}}{\text{sec}}$ → unit (speed)

→ T $\frac{m}{sec}$ ⟷ 10^{12} $\frac{m}{sec}$

II prefixes which made unit smaller :→

	prefixes	Value
(a)	deci ⟷	$10^{-1} = \frac{1}{10^1} = 0.1$
(b)	centi (c) ⟷	$10^{-2} = \frac{1}{10^2} = 0.01$
(c)	milli (m) ⟷	$10^{-3} = \frac{1}{10^3} = 0.001$
(d)	micro (μ) ⟷	$10^{-6} = \frac{1}{10^6} = 0.000001$

(e) nano (n) $\longleftrightarrow$ 10^{-9}

(f) pico (p) $\longleftrightarrow$ 10^{-12}

(g) femto (f) $\longleftrightarrow$ 10^{-15}

(h) atto (a) $\longleftrightarrow$ 10^{-18}

example :— meter $\longrightarrow$ unit (length)

$\longrightarrow$ centi meter $= 10^{-2}$ meter

example gram $\longrightarrow$ unit (mass)

$\longrightarrow$ centi gram $\longrightarrow$ 10^{-2} gram

$= 0.01$ gram.

$\longrightarrow$ micro gram $\longrightarrow$ 10^{-6} gram.

$\longrightarrow$ 10^{-9} sec $\longrightarrow$ n sec.

NOTE :$\longrightarrow$

** (a) 1 angstrum $= 1$ Å $= 10^{-10}$ meter.

(b) 1 fermi $= 1$ f $= 10^{-15}$ meter.

Engg. Mechanics

Application of prefixes :→

(a) 1 m = ---- cm

↓

L.H.S

$1\,m = \frac{1\ \text{centimeter}}{\text{(centi)}} = \frac{1\ cm}{10^{-2}}$

$= 10^{+2}\ cm = 100\ cm$

(b) 1 milli sec = ---- Sec

L.H.S

$1\ \text{millisec} = 1 \times 10^{-3}\ \text{sec}$

$= 0.001\ \text{Sec}$

Ans//

(c) 1 km = ---- meter (m)

L.H.S

$1\ km = 1 \times 10^{3}\ \text{meter}$

$= 1000\ \text{meter}$

Ans//

Q

$$1 \text{ newton} = 1 \ \frac{kg\,m}{sec^2} = \text{-----} \ \frac{g\,cm}{sec^2}$$

dyne

L.H.S

$$1 \frac{kg\,m}{sec^2} = 1 \times 10^{+3} \frac{g\,cm}{c\ sec^2}$$

$$= \frac{10^{+3}}{10^{-2}} \frac{g\,cm}{sec^2}$$

$$= 10^3 \times 10^2 \ \frac{g\,cm}{sec^2}$$

$$= 10^{3+2} \ \frac{g\,cm}{sec^2}$$

$$= 10^5 \ \frac{g\,cm}{sec^2}$$

Ans//

Q

$$1 \text{ Joule} = 1 \ N\text{-}m = 1 \ \frac{kg\,m^2}{sec^2} = \text{----} \ \frac{g\,cm^2}{sec^2}$$

L.H.S =

$$1 \frac{kg\,m^2}{sec^2} = 1 \times 10^3 \ \frac{g\ cm \times cm}{centi \times centi \times sec^2}$$

$$= \frac{10^3}{10^{-2} \times 10^{-2}} \frac{g\,cm^2}{sec^2}$$

$$= 10^3 \times 10^{+2} \times 10^{+2} \ \frac{g\,cm^2}{sec^2} = 10^7 \ \frac{g\,cm^2}{sec^2}$$

Ans//

Q $1\,J = \dfrac{Kg\,m^2}{Sec^2}$

$\alpha \rightarrow$ alph
$\beta \rightarrow$ Beta
$\gamma \rightarrow$ gamma.

If unit of mass is α Kg
and " " " length is β m.
and " " " Time is γ sec.
then prove that $1\,J = \alpha^{-1}\beta^{-2}\gamma^{2}$ new system of unit.

Solution :-

Given

$\because\ 1\,J = 1\,\dfrac{Kg\,m^2}{Sec^2}$

$= 1 \times \dfrac{\alpha\,Kg\ \beta m \times \beta m}{\alpha \times \beta \times \beta \times \gamma Sec \times \gamma Sec}$ new system of unit.

$= \alpha^{-1}\beta^{-2}\gamma^{2}$ new system of unit proved

Engg. Mechanics

VECTOR :→

→ Those physical quantities which have magnitude, a definite direction and follow law of parallelogram are called vector quantities.

example:— Displacement, Velocity, acceleration, Force, Impulse, momentum, moment, electric field, Magnetic field etc.

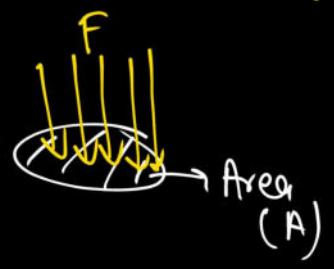
→ Those physical quantities which have magnitude only are called Scalar quantities.

example:— Mass, length, time, area, Temperature, volume density, work, Current, pressure etc.

NOTE:—

(a) ** Current has both magnitude and direction but does not follow of parallelogram, hence it is a scalar quantity.

(b) pressure(P) = $\frac{\text{force}}{\text{Area}} = \frac{F}{A}$

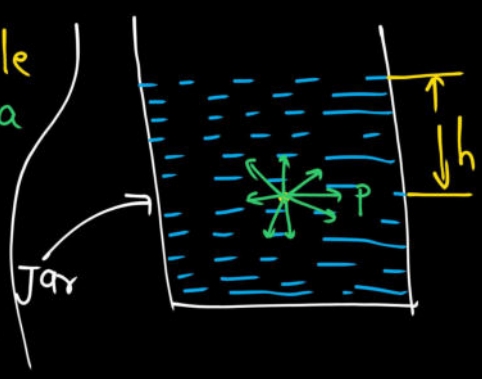


→ law of Hydrauestatics:—

(pascle's law):—

pressure has multiple direction at a time. hence pressure is scalar quantity.

P = pressure



Representation of a vector :→

ⓐ A vector is geometrically represent-ed by an arrow (↗).

ⓑ A vector is denoted by either a bold letter or a letter and an arrow on it.

ⓒ The length of the arrow show magnitude of the vector.

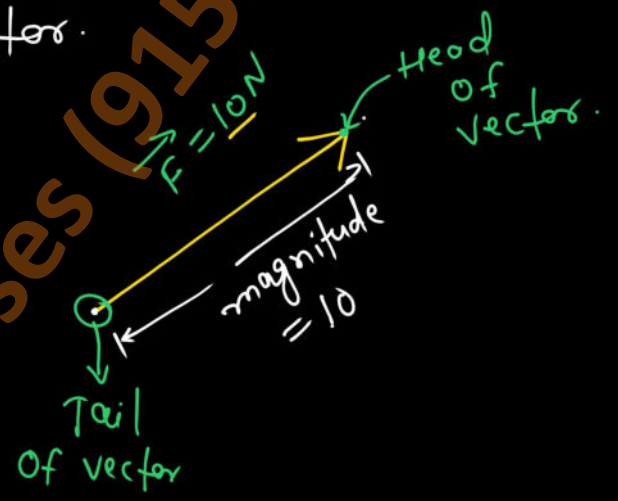
ⓓ Orientation of arrow shows direction of the vector.

example

$\vec{F} = 10\,N$

↓

Force.



NOTE :→

** ⓐ Magnitude of a vector is always a positive real number.

ⓑ $|\vec{A}|$ = Magnitude of $\vec{A}$.

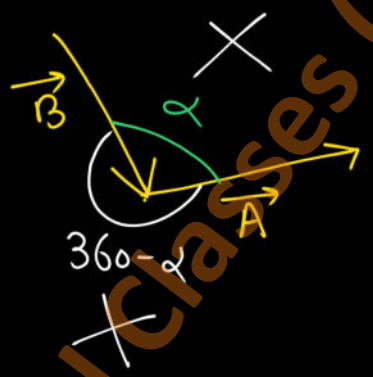
mode

example $\vec{F} = 10\,N$

$|\vec{F}| = 10$ newton.

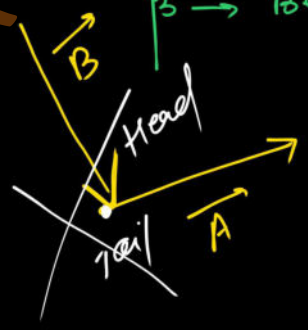
Engg. Mechanics

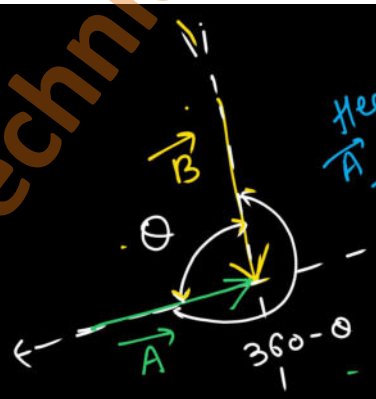
Angle between two vectors.



$\alpha \rightarrow$ alpha

$\beta \rightarrow$ Beta.



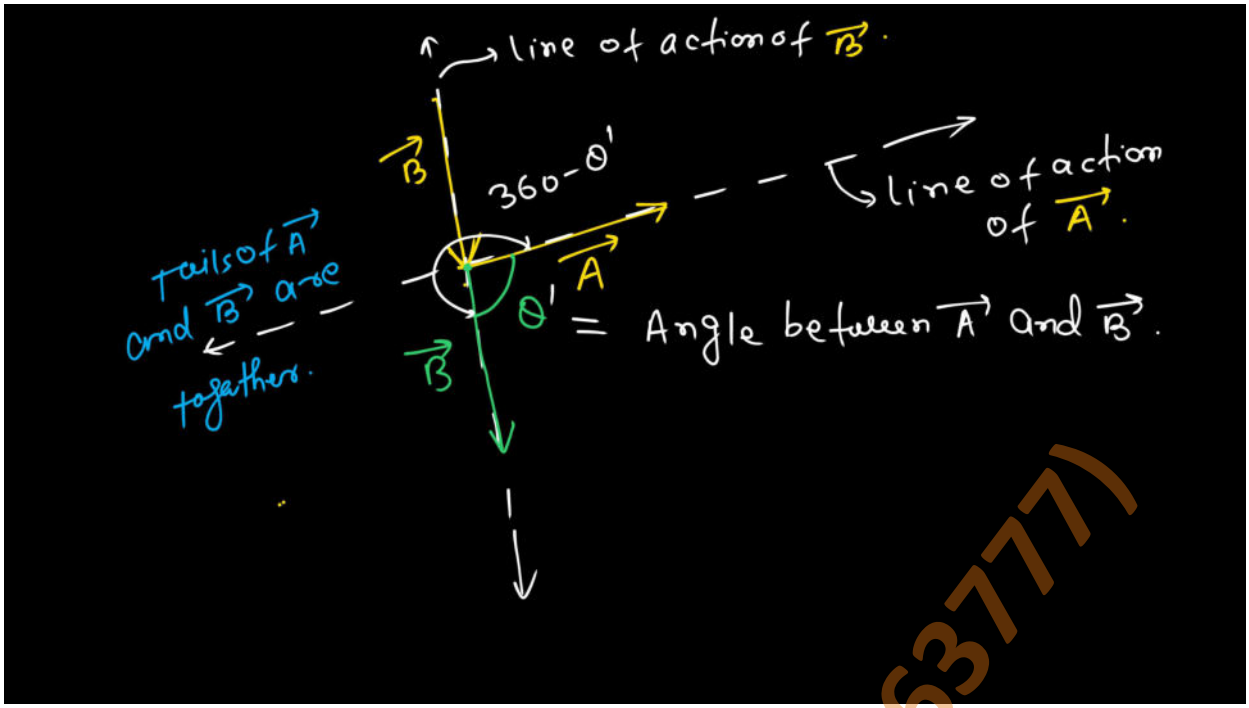


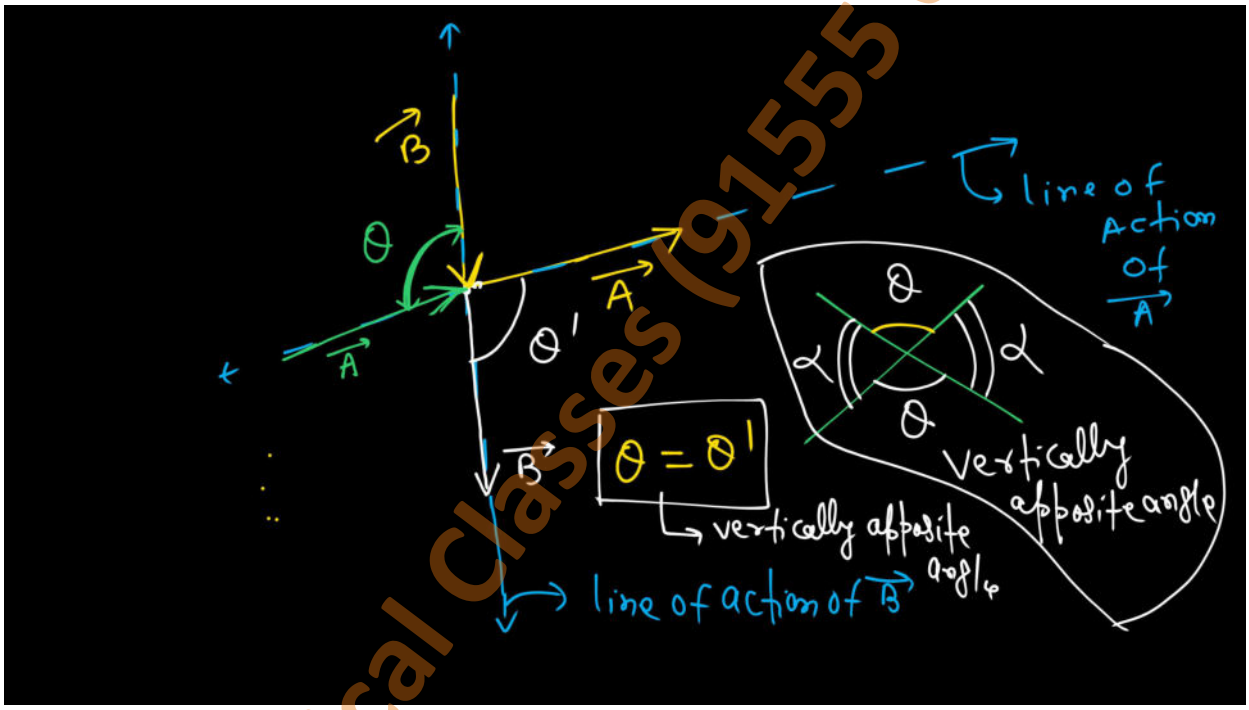
Head of $\vec{A}$ and Head of $\vec{B}$ are together.

$\rightarrow$ Line of Action

θ = Angle between $\vec{A}$ and $\vec{B}$.

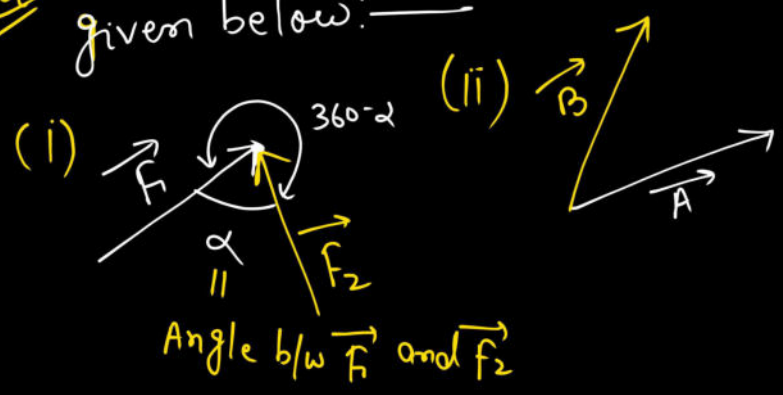
$\rightarrow$ line of action of $\vec{B}$





Home work :→

Q Show the angle between two vectors given below:—



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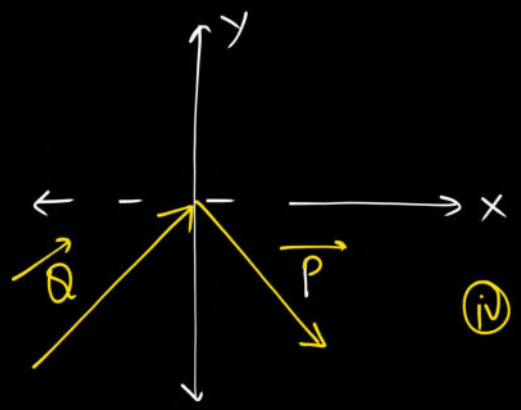
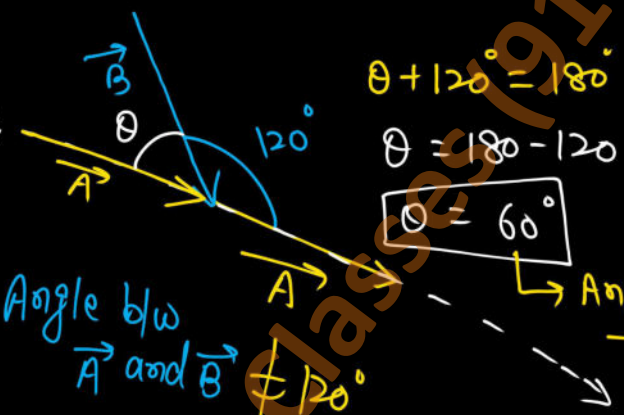


(iv)

Engg. Mechanics

Q.



line of action of $\vec{A}$

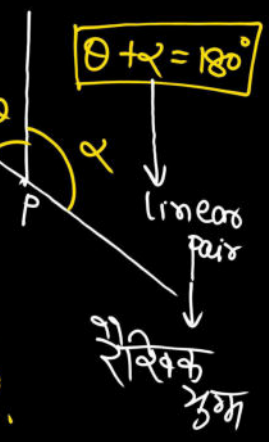
Angle b/w $\vec{A}$ and $\vec{B}$ $\neq 120°$

$\theta + 120° = 180°$

$\theta = 180 - 120$

$\boxed{\theta = 60°}$

↳ Angle b/w $\vec{A}$ and $\vec{B}$.

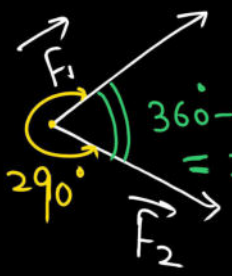


$\boxed{\theta + \alpha = 180°}$

↓ linear pair

↓ रैखिक युग्म

Q.

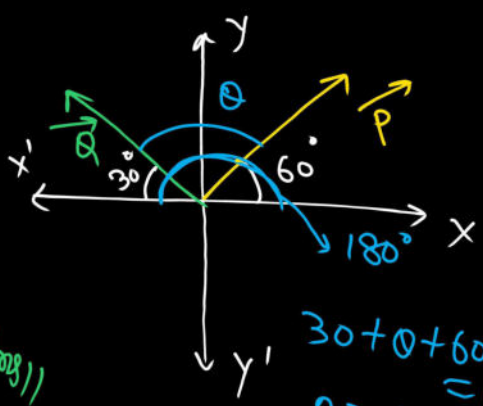


$360° - 290°$

$= 70°$ Ans

↳ Angle b/w $\vec{F_1}$ and $\vec{F_2}$

Q

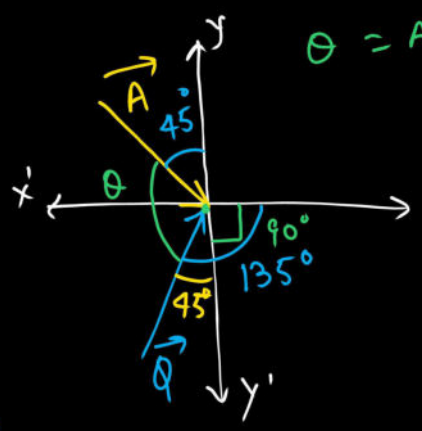


$30 + \theta + 60 = 180°$

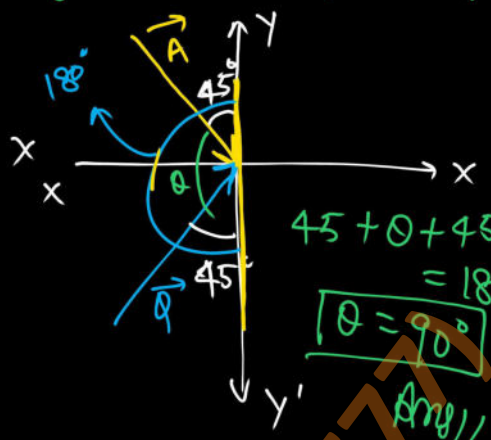
$\theta = 180 - 30 - 60$

$\boxed{\theta = 90°}$ Ans

Q



θ = Angle between $\vec{A}$ and $\vec{Q}$

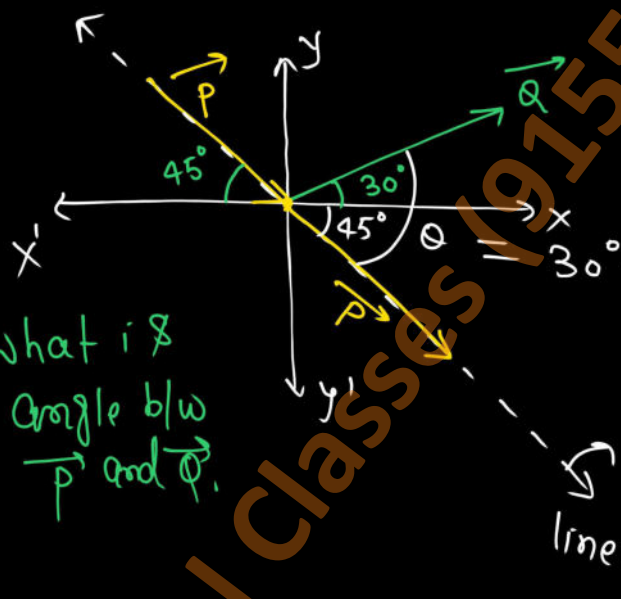


$45 + \theta + 45 = 180°$

$\boxed{\theta = 90°}$

Ans//

Q

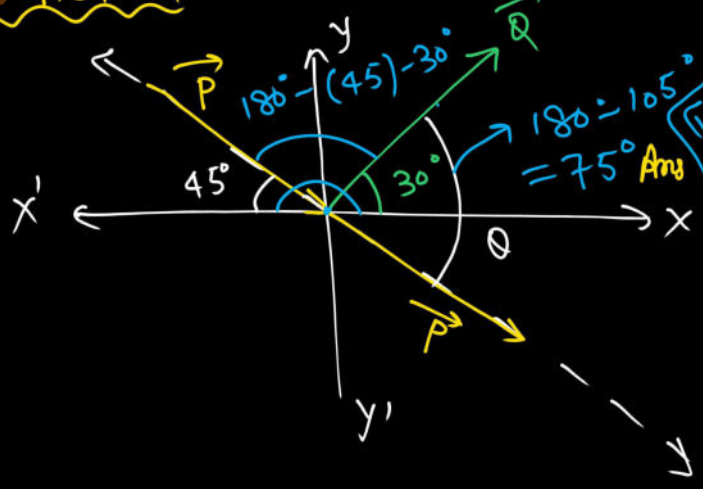


$\theta = 30° + 45° = 75°$

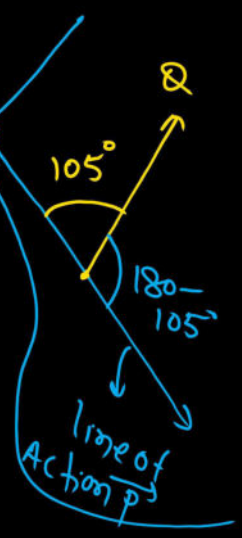
Ans//

What is angle b/w $\vec{P}$ and $\vec{Q}$.

2nd Method :-



$180 - 105°$
$= 75°$ Ans

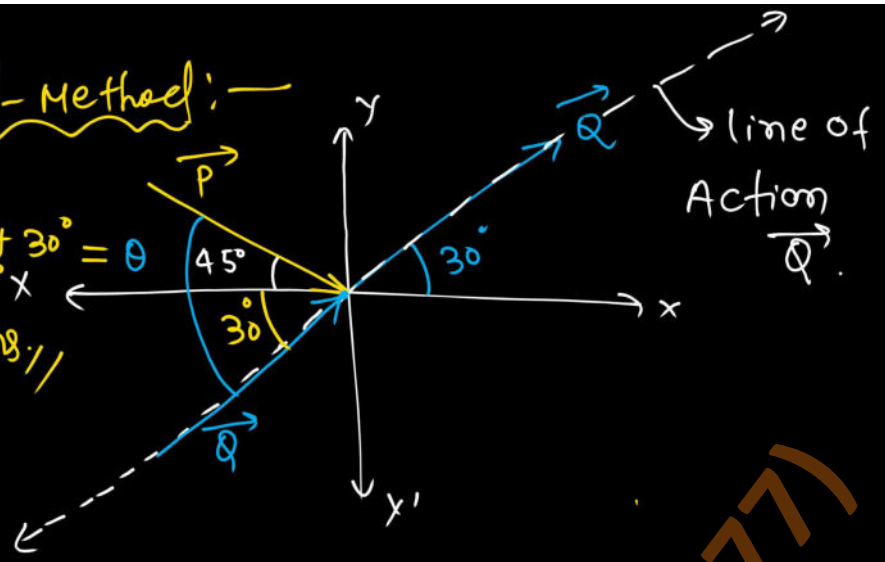


3rd Method :—

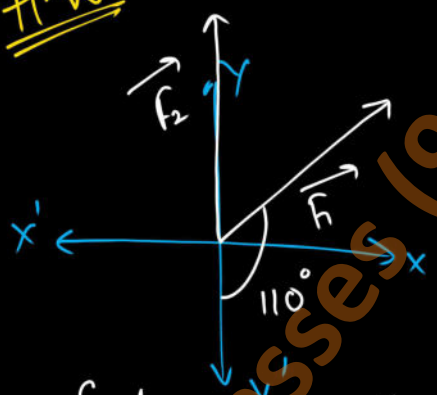
$45^\circ + 30^\circ = \theta$

$= 75^\circ$ ✗

Ans.//

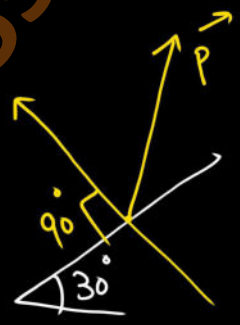


Q H.W



find angle b/w $\vec{F_1}$ and $\vec{F_2}$.

Q



Find angle between $\vec{P}$ and $\vec{Q}$.

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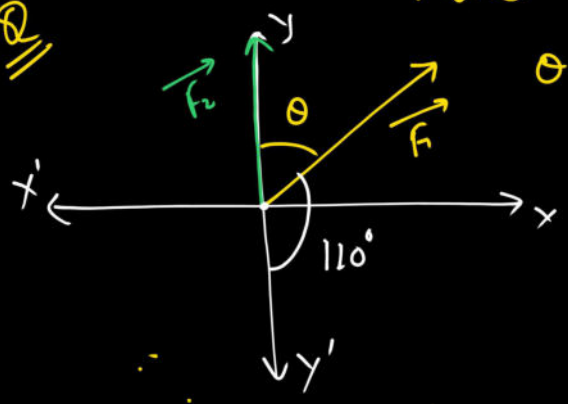
Q



θ = angle b/w $\vec{F}$ and $\vec{F_2}$.

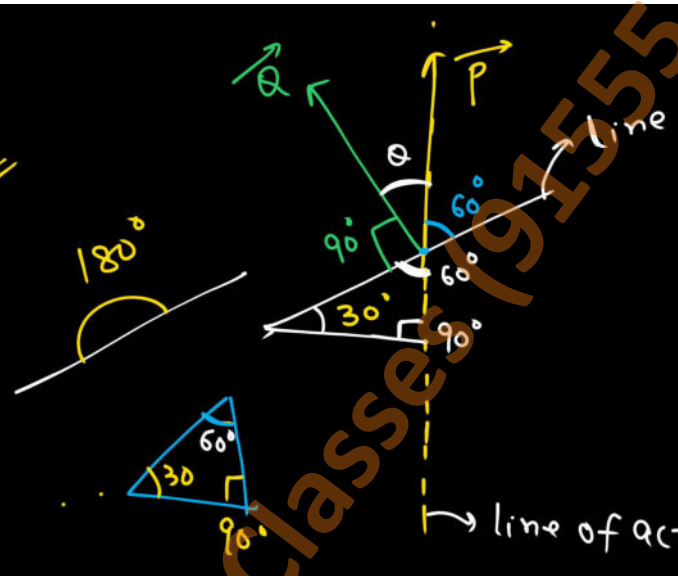
$\because 110° + \theta = 180°$

$\Rightarrow \theta = 180° - 110°$ [linear pair]

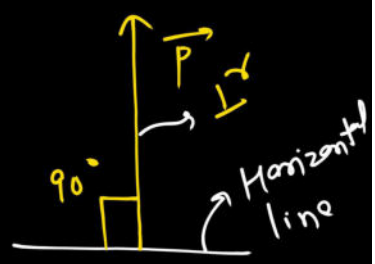
$\boxed{\theta = 70°}$

Ans//

Q



θ = Angle between $\vec{P}$ and $\vec{Q}$.



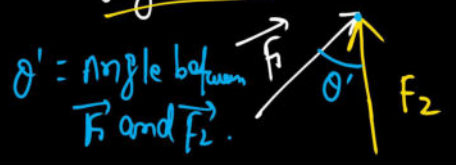
$\because \theta + 60° + 90° = 180°$

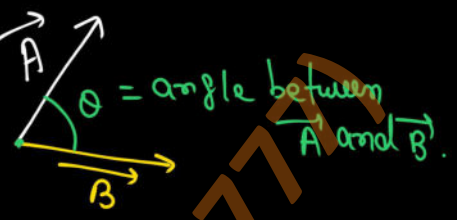
$\theta = 180 - 60 - 90$

$\boxed{\theta = 30°}$ Ans//

Angle between two vectors :→

→ Angle between two vectors is that **Small angle** formed when **either** tails **or** heads of the two vectors are **together** at a point.





*** # TYPES of VECTORS :→

I UNIT VECTOR :→

→ Those vectors whose **magnitude** is unity (1) are called unit vectors.

→ A unit vector is **denoted by a letter and a cap on it**. example: $\hat{A}$, $\hat{a}$, $\hat{i}$, $\hat{j}$, $\hat{k}$

→ A unit vector is **use to give direction of a vector**.

→ A unit vector ($\hat{P}$) corresponding to a vector $\vec{P}$ is give by :—

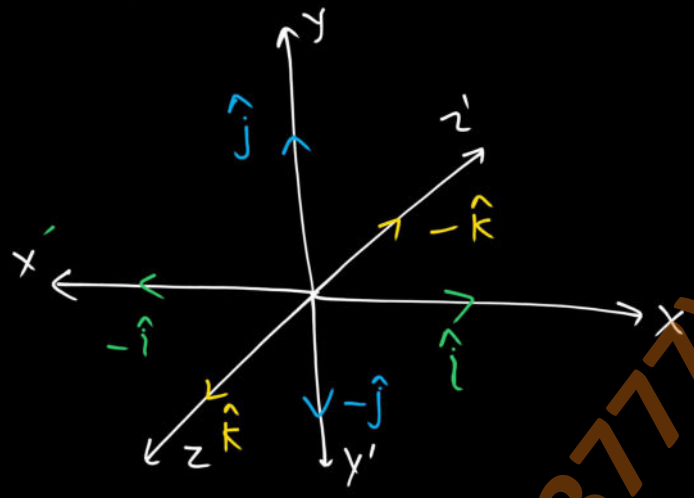
**

$$\boxed{\hat{P} = \frac{\vec{P}}{|\vec{P}|}}$$

$\hat{P}$ → unit vector $\hat{p}$.

$|\vec{P}|$ → Magnitude of $\vec{P}$.

Some useful standard unit vectors :→



$\hat{i} \rightarrow$ Direction along + x-axis

$\hat{j} \rightarrow$ " " + y-axis

$\hat{k} \rightarrow$ " " + z-axis.

$-\hat{i} \rightarrow$ " " − x-axis

$-\hat{j} \rightarrow$ " " − y-axis

$-\hat{k} \rightarrow$ " " − z-axis.

NOTE :—

$|\hat{i}| = 1$

$|\hat{j}| = 1$

$|\hat{k}| = 1$

$|-\hat{i}| = 1$

$|-\hat{j}| = 1$

$|-\hat{k}| = 1$

$|\hat{p}| = 1$

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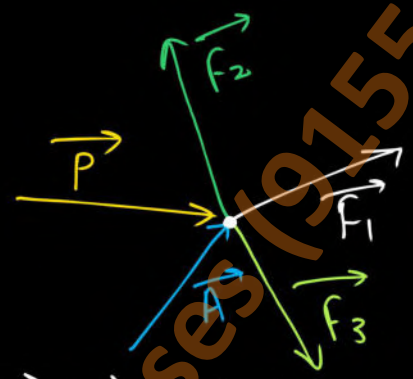
II Concurrent vectors :→

→ All those vectors whose of line of actions passes through a single point are called concurrent vectors.

→ In other words. All those vectors which have one point in common are called concurrent vectors

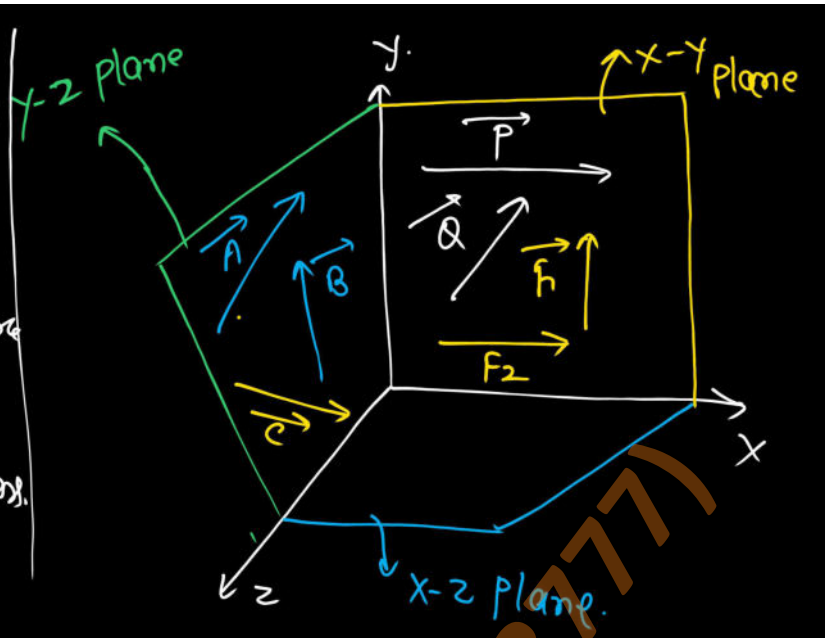


$\vec{P}$, $\vec{A}$, $\vec{F_1}$, $\vec{F_2}$ and $\vec{F_3}$ are concurrent vectors.

III Coplanar vectors :→

→ All those vectors which lie on a single (same) plane are called coplanar vectors.

$\vec{P}$, $\vec{Q}$, $\vec{F_1}$ and $\vec{F_2}$ all are leing in x-y plane therefore they are Coplanar vectors.



example, $\vec{A}$, $\vec{B}$ and $\vec{C}$ are coplanar vectors, because they are lie in same y-z plane.

$\vec{A}$, $\vec{B}$, $\vec{C}$ → y-z plane

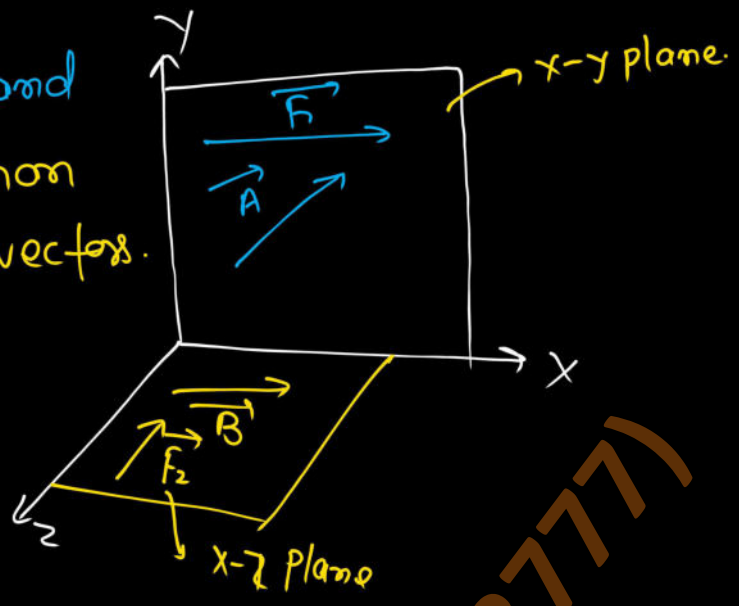
$\vec{F_1}$ & $\vec{F_2}$ → x-y plane

Non-coplanar vectors.

IV Non-coplanar vectors:→

All those vectors which do not lie on a same (single) plane are called non co-planar vectors.

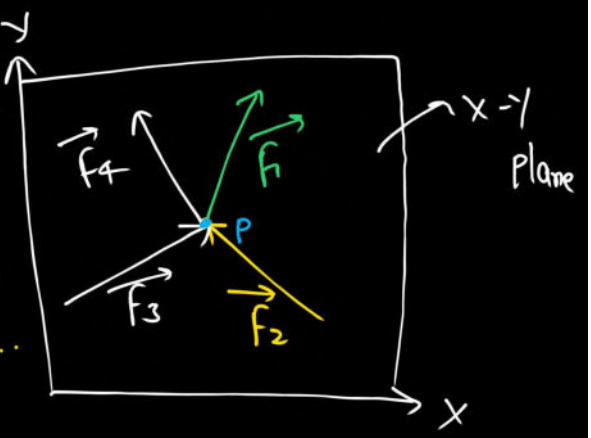
$\vec{F_1}$, $\vec{F_2}$, $\vec{A}$ and $\vec{B}$ are non coplanar vectors.



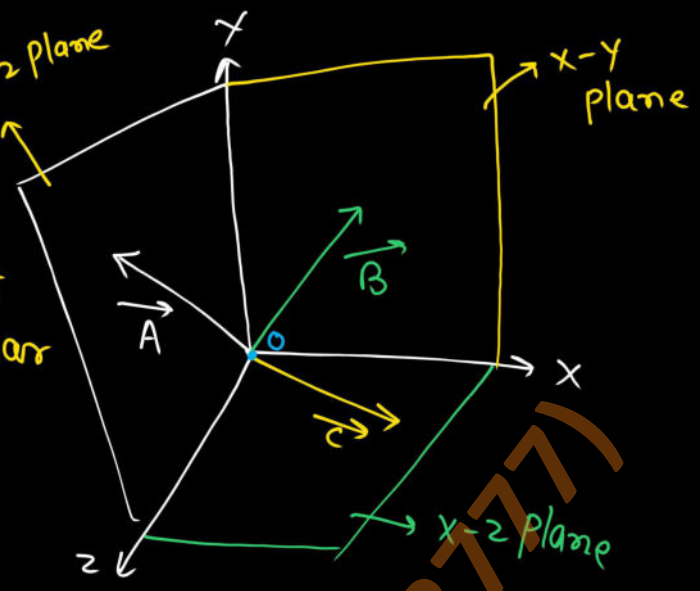
Q. Concurrents vectors may be coplanar vectors and non-coplanar vectors (True/False)

Ans:— TRUE

→ Here $\vec{F_1}$, $\vec{F_2}$, $\vec{F_3}$, and $\vec{F_4}$ are concurrent and coplanar vectors..



Here $\vec{A}$, $\vec{B}$ and $\vec{C}$ are concurrent but non-coplanar vectors.



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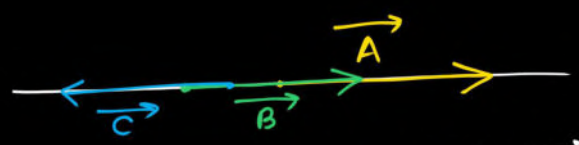
IV. Collinear vectors :→

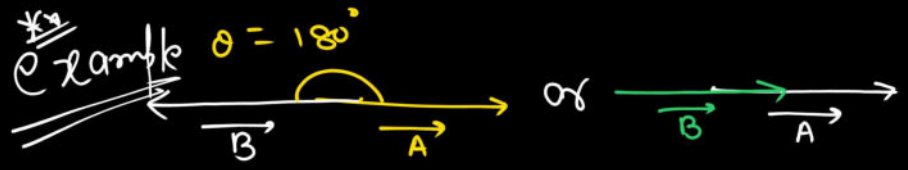
→ All those vectors whose arrow lie on a same line, are called Collinear vectors.



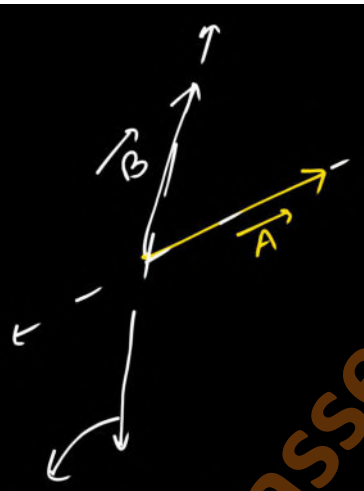
All three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ are called Collinear vectors.

NOTE* → Angle between two collinear vectors is either 0° (same dir^n) or 180° (opposite direction)

Example $\theta = 180^\circ$



θ = Angle between two collinear vector $\vec{A}$ & $\vec{B}$



line of $\vec{A}$.

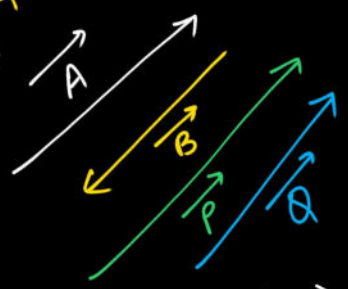
$\vec{A}$ and $\vec{B}$ are non collinear vector

line of $\vec{B}$.

V Parallel vectors :→ All those vectors whose line of action are parallel to each other are called parallel vectors.

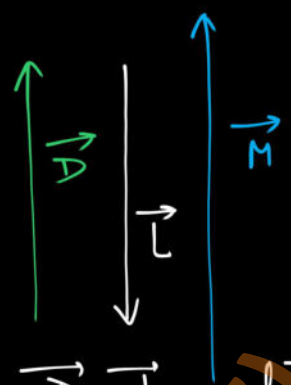
example:

(i)



$\vec{A}$, $\vec{B}$, $\vec{P}$ and $\vec{Q}$ are parallel (||) vectors.

(ii)



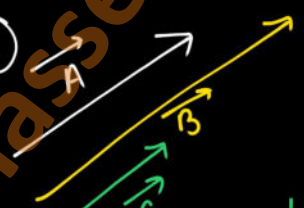
$\vec{D}$, $\vec{L}$ and $\vec{M}$ are parallel (||) vectors.

(a) Like parallel vectors :→

→ All those parallel vectors which same direction are called like parallel vectors.

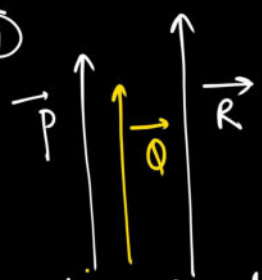
example:

(i)



$\vec{A}$, $\vec{B}$ and $\vec{C}$ are like || vectors.

(ii)



like || (parallel) vect.

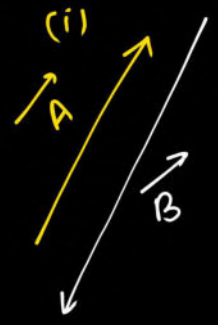
** Note :→ Angle between two like parallel vectors is zero deg. (0°)

(b) Unlike/anti parallel vectors :→

→ Two parallel vectors are called anti parallel vectors when their directions are opposite to each other.

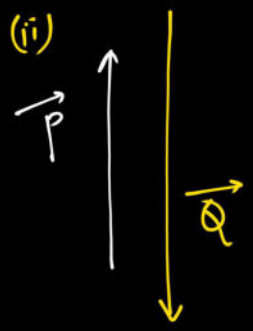
example

(i)



$\vec{A}$ and $\vec{B}$ are unlike/anti parallel vectors.

(ii)



$\vec{P}$ and $\vec{Q}$ are anti parallel vectors.

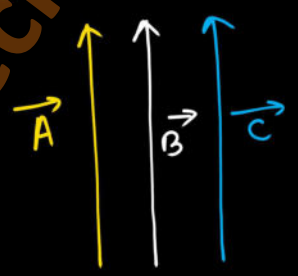
Note :- Angle between two unlike or anti parallel vectors is always 180°.

VI Equal vectors :→

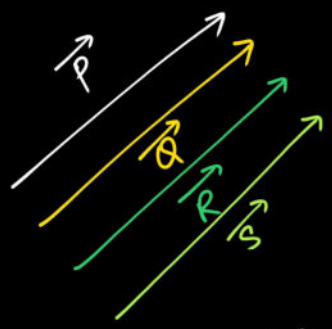
→ All those vectors which have same magnitude and same direction are called equal vectors.

example

(i)



Here $\vec{A}$, $\vec{B}$ and $\vec{C}$ are equal vectors



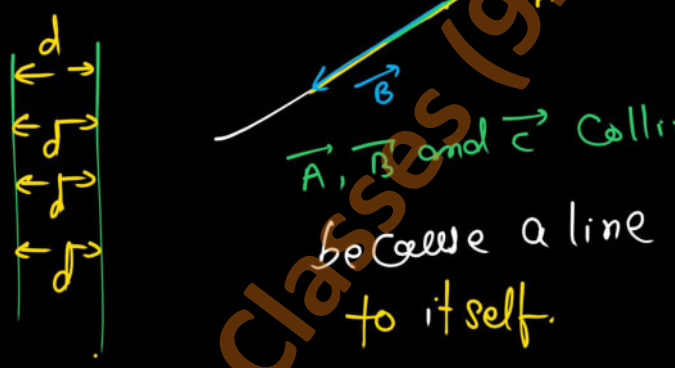
$\vec{P}$, $\vec{Q}$, $\vec{R}$ and $\vec{S}$ are equal vectors.

NOTE: → Angle between two equal vectors is always 0° (zero deg.).

Q: All collinear vectors are parallel vectors (True/False).

Ans:- True (T).

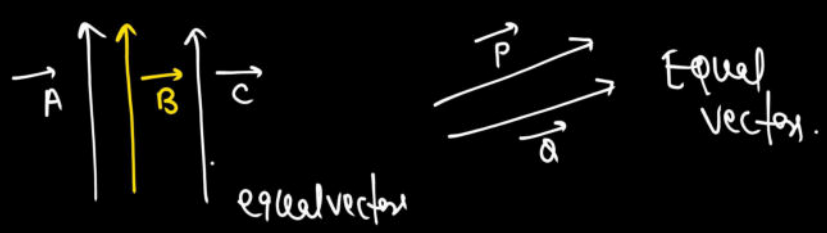
explanation



$\vec{A}$, $\vec{B}$ and $\vec{C}$ collinear and parallel because a line is also parallel to itself.

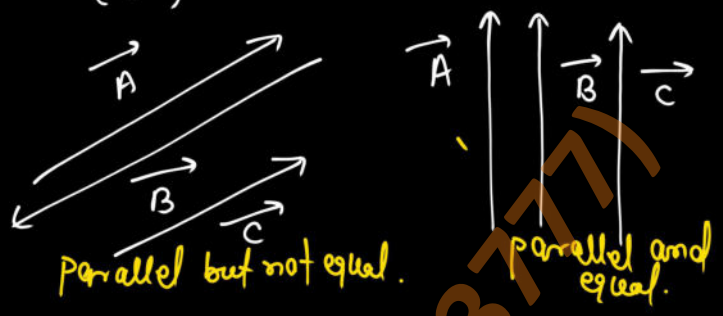
Q: All Equal vectors are always parallel vectors (True/False)

Ans:- True (T)



Q All parallel vectors are always equal vectors (True/False)

Ans :- False (F)



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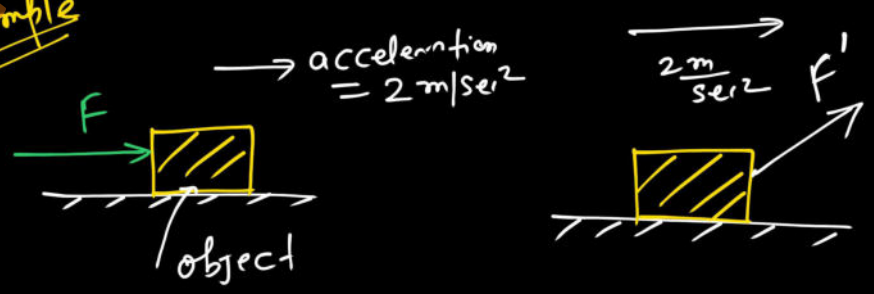
VII :- Equivalent vectors :→

→ Two vectors are said to be a equivalent vectors when there external effect on an object is same.

example



F → force effect → $\frac{2m}{sec^2}$ = Same

∴ Force F and F' are equivalent force.

Note :→

(i) Force is a "Sliding vector".

(ii) 'Moment" is a "rotational free vector."

FORCE :→

→ Force is an "action" of one object/body on to another body by coming in contact or without contact with the body.

(a) Contact force :→ When force acted between two Object in contact is called contact force.

example :— Friction force, Tension, Spring force etc.

(b) **Non contact force :→**

when force is acted between two objects which are not in contact then force is called non contact type force.

example:- Gravitational force, Magnetic force, electrostatic force.

Characteristics of a force :→

followings are the important characteristics of a force :→

(a) **Magnitude :→** A force has a positive numerical value which gives its magnitude.

example, 5 N, 50 N, 100 N

Mg.

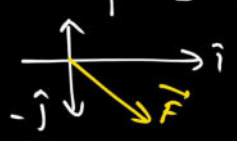
(b) **Direction :→**

→ force is a vector physical quantity so it has a definite direction

example:— $\vec{F} = 3\hat{i}$ newton

→ direction along +ve x-axis.

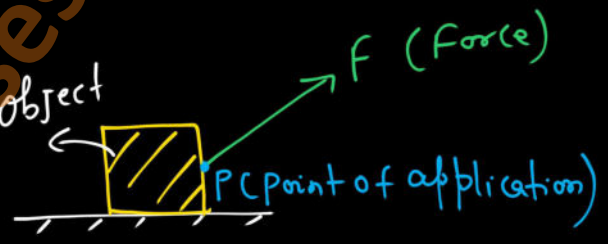
$\vec{F} = 5\hat{i} - 2\hat{j}$



(c) point of application :→

→ A force acts on an object always has a point of application on the object.

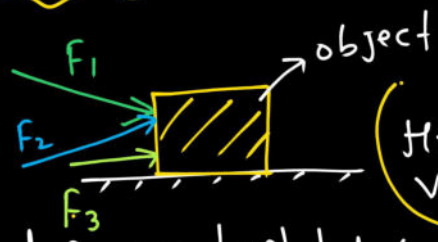
example



(d) Nature of force :→

There are two nature of force :—

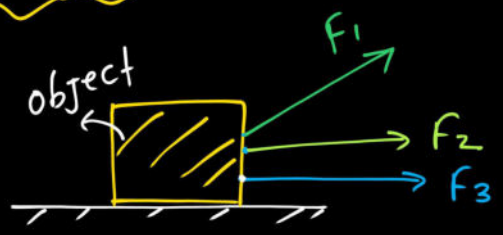
(i) push type force :→



Head of force vector is on the object.

F_1, F_2 and F_3 are push type force.

(ii) Pull type force :→



F_1, F_2 and F_3 are of pull type force.

Tail of the force vector is on the object

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Effects of force :→

followings are the important effects of force :—

(a) Force can change or tries to change the state of an object.

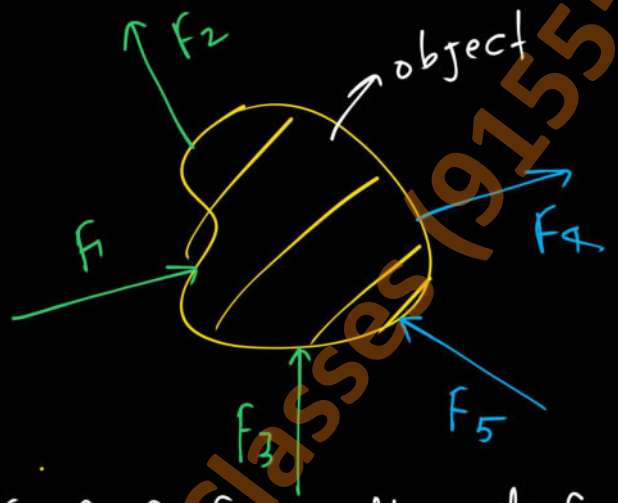
(b) Force can move an object which is in rest and can bring an object in the state of rest which is in motion.

(c) Force can accelerate or deccerate the motion of an object.

(c) Deforming forces can produce internal effect in the objects like stress and strain.

System of forces :→

→ when there are number of forces acting on an object or at a point, then Simultaneously they are called system of forces.

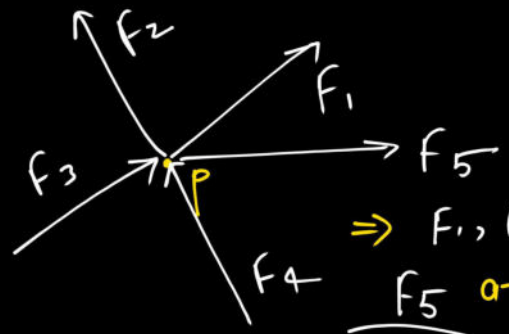


F_1, F_2, F_3, F_4, F_5 Simultaneously form system of forces.

Types of System of forces :→

I Concurrent forces :→

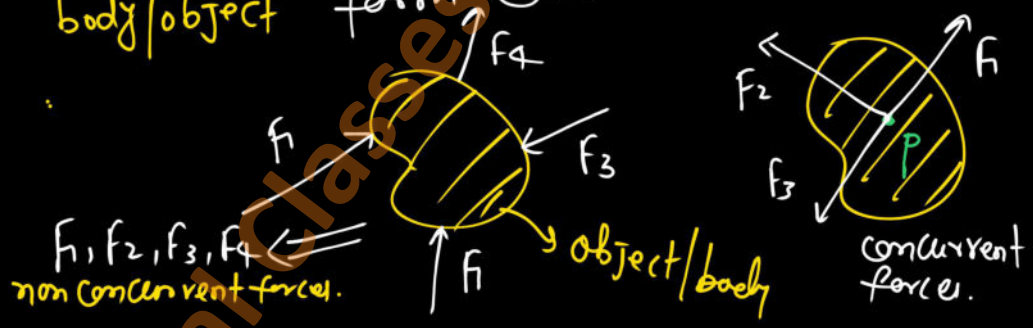
All those force whose line of action passes through a Common point (Same point) are called concurrent forces.



⇒ F_1, F_2, F_3, F_4 and F_5 are concurrent forces.

Note:→ (a) forces acting on a "particle" always form concurrent forces.

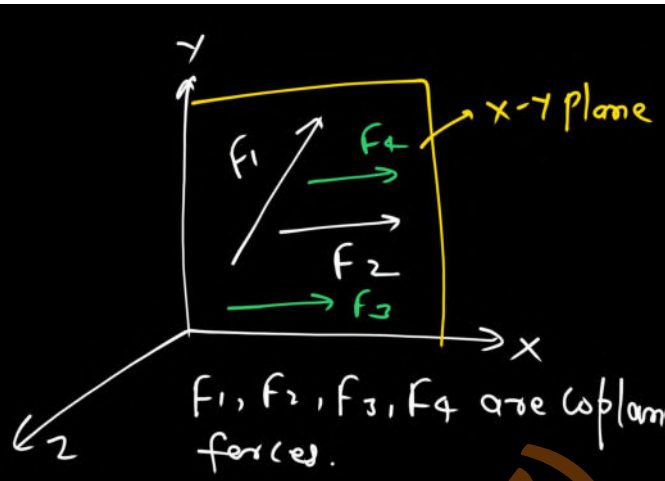
(b) Forces acting on a body need not neccessarily form concurrent forces but forces acting at a point on the body/object form concurrent forces.



II Coplanar forces :→

→ All those forces whose lines of action lie on a single plane are called coplanar forces.

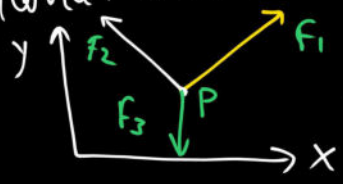
→ In Engg. mechanics subject we have to deal with co-planar forces only.



F_1, F_2, F_3, F_4 are coplanar forces.

III Coplanar and concurrent forces :→

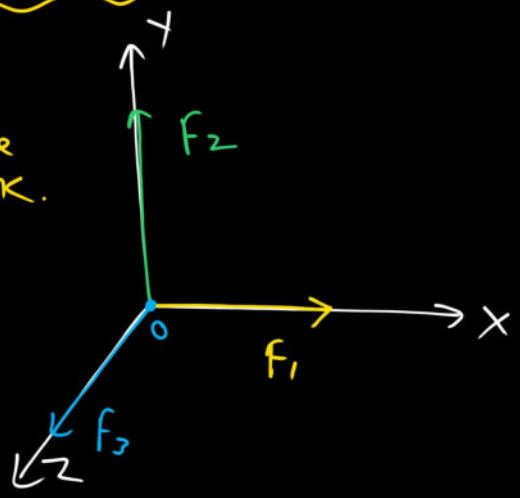
→ All those forces whose line of action lie on same plane and also passes through a single common point are called coplanar and concurrent forces.



F_1, F_2, F_3 are coplanar and concurrent forces.

IV Non coplanar and concurrent forces :→

Definition → Home work.

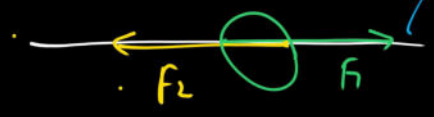


V Collinear forces :→

VI Collinear and coplanar forces :→

VII Collinear and concurrent forces :—

Not possible



H.W definitions

VII Parallel forces : → like ∥ forces, → unlike ∥ for

VII parallel coplanar forces :—

IX parallel and non coplanar forces :→

X Equal forces.

XI Equivalent forces.

Engg. Mechanics

Q Rigid bodies do not experience _____

(a) Deformation (b) Translation

(c) Rotation (d) Acceleration.

Q. _____ is a scalar quantity.

(a) Force (b) Velocity (c) Temperature

(d) Displacement

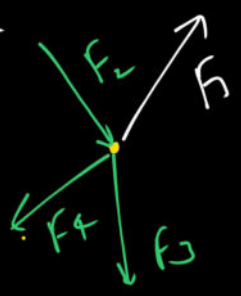
Q. A concurrent force system is one in which all forces _____

(a) act at the same point

(b) Act in the same direction

(c) Act along parallel lines

(d) Act in opposite direction.



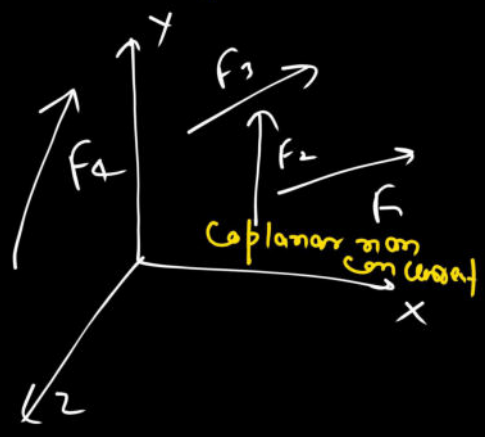
Q. The non concurrent forces may be _____

(a) coplanar

(b) Non coplanar

(c) Both (a) and (b)

(d) None of these.



Q Collinear forces - - - - -

(a) are concurrent at a point

(b) Are parallel to each other

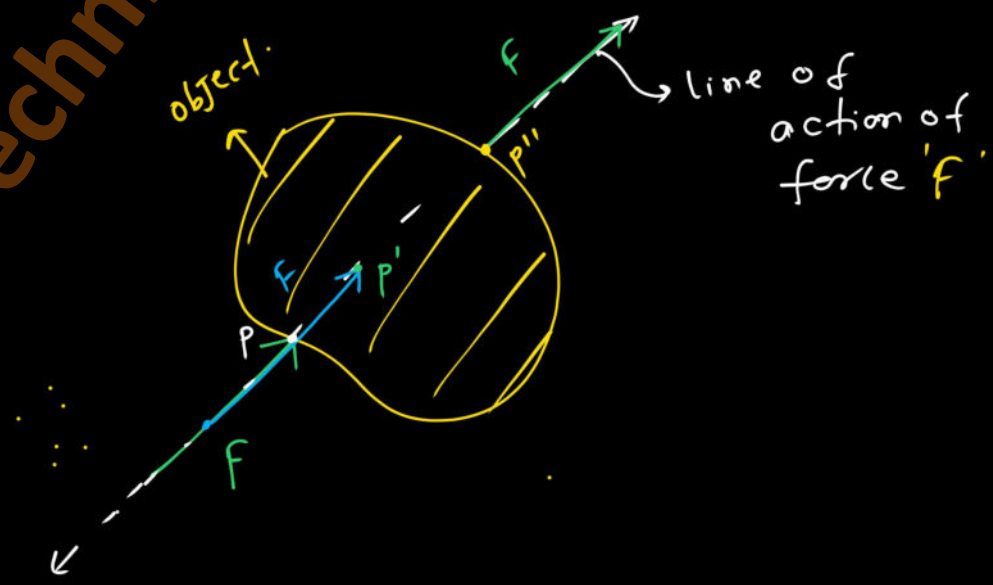
(c) lie on the same line

(d) None of these.

Collinear forces.

principle of transmissibility of force :→

→ principle of transmissibility of force states that the state of an object can be maintained if a force acting on the object at a point can slide or transmit along its line of action at some other point on the object.

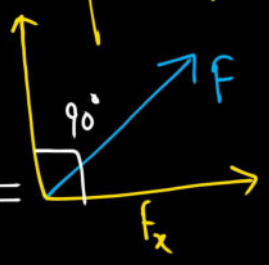


V.V.I

Resolution of 'a' force :→

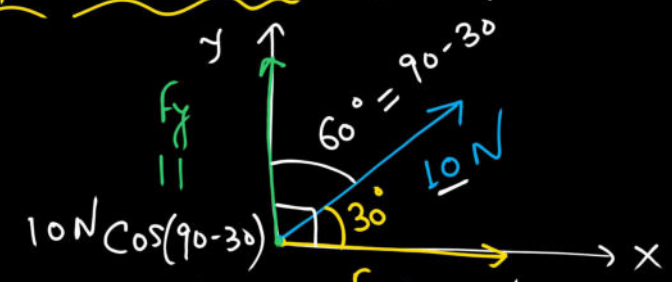
→ Resolution of 'a' force is the process of splitting that force into number of parts (component) which are simultaneously equivalent to that single force.

→ If a force is resolved in two components which are perpendicular to each other, then the components are called rectangular components or orthogonal components.



Here F_x, and F_y are rectangular or orthogonal component of force 'F'.

Method of resolution :→



$F_y = 10N\cos(90-30)$

$= 10N \times \cos 60$

$= 10N \times \frac{1}{2}$

$= 5N$

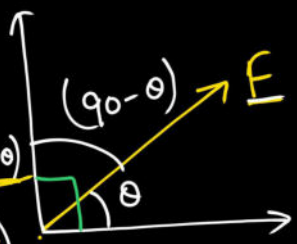
$F_x = 10N\cos 30°$

$= 10N \times \frac{\sqrt{3}}{2}$

$= 5\sqrt{3}\ N$

example

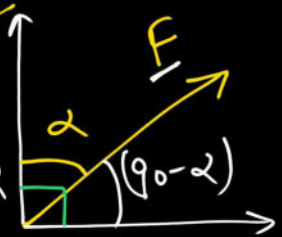
$f_2 = F\cos(90-\theta) = F\sin\theta$



$f_1 = F\cos\theta$

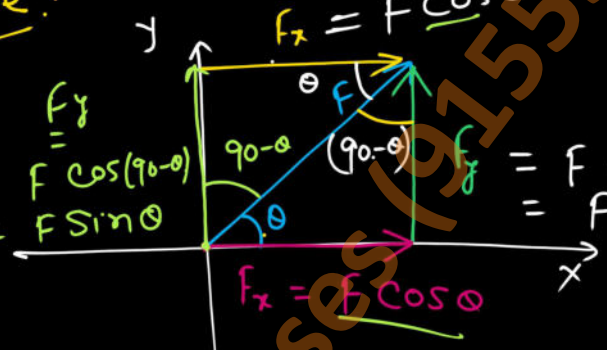
example

$f_2 = F\cos\alpha$



$f_1 = F\cos(90-\alpha) = F\sin\alpha$

Example:

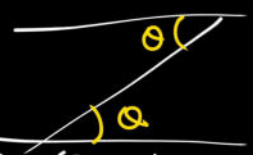


$f_x = F\cos\theta$

$f_y = F\cos(90-\theta) = F\sin\theta$

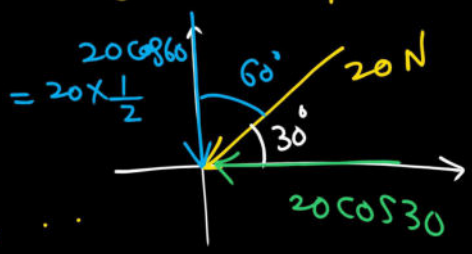
$f_y = F\cos(90-\theta) = F\sin\theta$

$f_x = F\cos\theta$



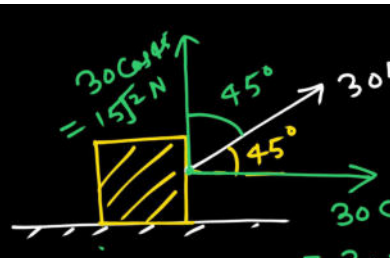
Engg. Mechanics

Q: Resolve the followings force in rectangular components: →



$\sin 0 = 0$	$\cos 0 = 1$
$\sin 45 = \frac{1}{\sqrt{2}}$	$\cos 45 = \frac{1}{\sqrt{2}}$
$\sin 30 = \frac{1}{2}$	$\cos 30 = \frac{\sqrt{3}}{2}$
$\sin 60 = \frac{\sqrt{3}}{2}$	$\cos 60 = \frac{1}{2}$
$\sin 90 = 1$	$\cos 90 = 0$

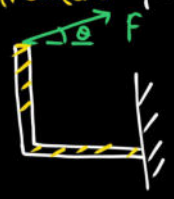
Q



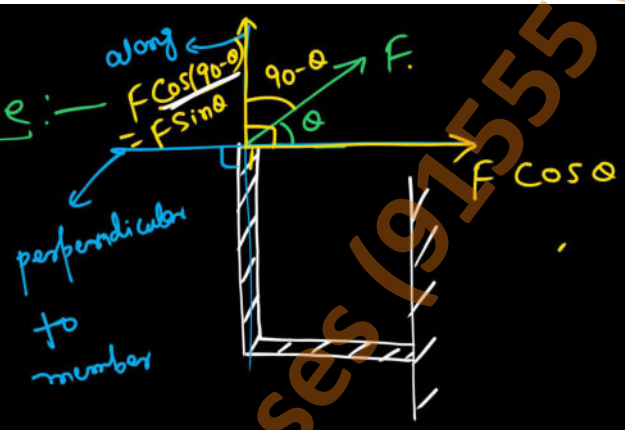
30N (pull type and "inclined" of 45° with horizontal)

$30\cos 45 = 30 \times \frac{1}{\sqrt{2}} = 15\sqrt{2}$ N

Q Resolve the following force along and perpendicular to the member it act on.



Solve :—



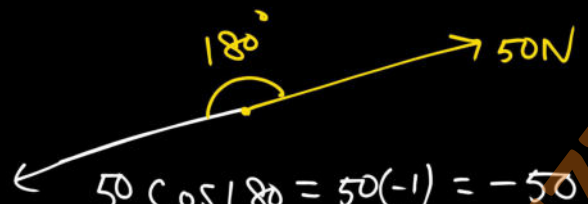
Q What will be the component of a force of magnitude 50N along its own direction.

(a) 25N (b) 12.5N (c) 50N (d) 100N.

$50\cos 0^\circ = 50 \times 1 = 50$ N

Q What will be the "Component" of a force of magnitude 50N in a direction opposite to direction of force.

(a) 25N (b) 12.5N (c) −50N (d) −100N



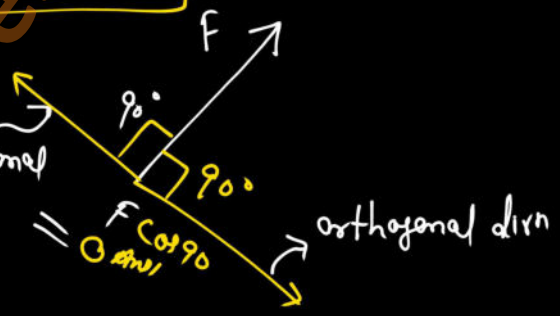
$50 \cos 180 = 50(-1) = -50$ Ans

Q : What will be the component of a force of magnitude 'F' in a orthogonal direction.

perpendicular

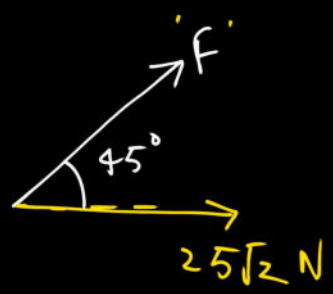
Note :- orthogonal → '90°'

Sol^n :- $F \cos 90°$
$= 0$ Ans



Q If component of a force in horizontal direction $25\sqrt{2}$ N, and the force made 45° with horizontal, Find the magnitude of force.

Solution :- let 'F' is the magnitude of force.



$\therefore$ Horizontal component $= F\cos 45$

$= 25\sqrt{2}$ (given)

$\Rightarrow F\cos 45 = 25\sqrt{2}\ N$

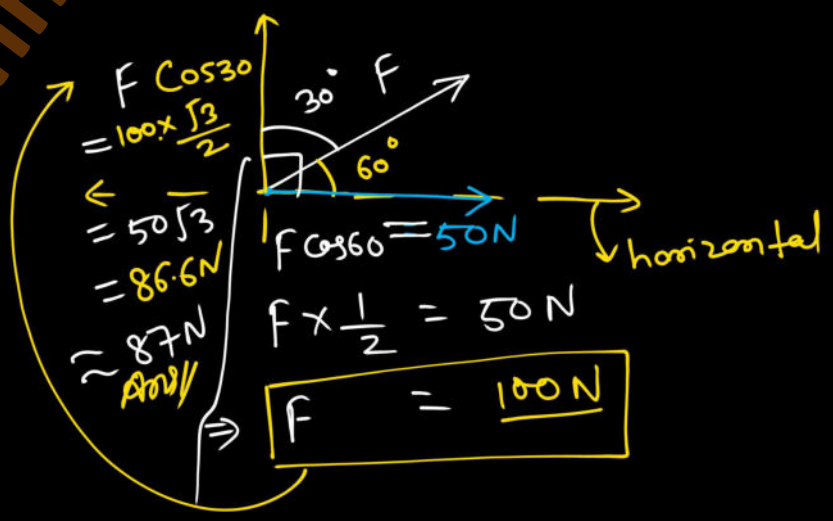
$\Rightarrow F \times \frac{1}{\sqrt{2}} = 25\sqrt{2}$

$F = 25\sqrt{2} \times \sqrt{2} = 50\ N$ Ans//

Engg. Mechanics

Q A force is 60° inclined to horizontal if the rectangular components in horizontal direction is 50 N, then magnitude of force in vertical direction is

(a) 25N (b) 87 N (c) 100N (d) 75N.



$F \times \frac{1}{2} = 50\ N$

$\Rightarrow \boxed{F = 100\ N}$

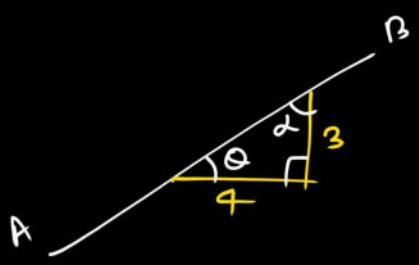
Q. Resolve the following force of magnitude 50N along the line 'AB' and perpendicular (⊥) to line 'AB' as shown below.

Ans//

Note ** (a)

(b)

(c)



$\tan\alpha = \frac{4}{3}$

α → line AB inclined with vertical line

$\tan\theta = \frac{3}{4}$

$\theta = \tan^{-1}\left(\frac{3}{4}\right) = 37°$

= line AB inclined with 'θ' to horizontal.

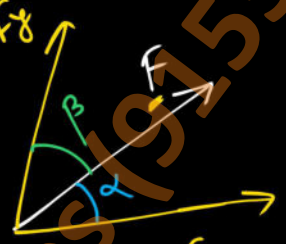
Q. F_x and F_y are the component of a force 'F' making an angle 'α' and 'β' with F_x and F_y respectively. If $\alpha < \beta$, then

(a) $F_x = F_y$ (b) $F_x > F_y$ (c) $F_x < F_y$

(d) None of these.

explanation

$F\cos\beta = F_y$



$F\cos\alpha = F_x$

Given

$\alpha < \beta$

$\cos\alpha > \cos\beta$

$\Rightarrow F\cos\alpha > F\cos\beta$

Ans $F_x > F_y$

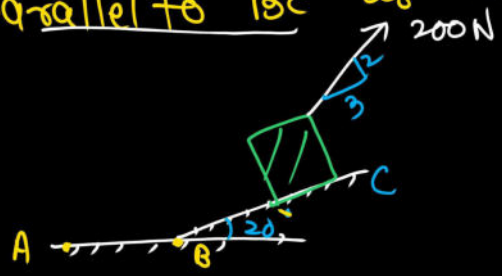
$30^\circ < 60^\circ$

$\cos 30 > \cos 60$

$\boxed{\frac{\sqrt{3}}{2} > \frac{1}{2}}$

Engg. Mechanics

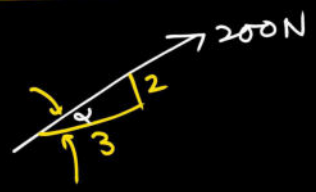
Q. Resolve the force F = 200 N into a component perpendicular to 'AB' and a component parallel to 'BC' as shown in fig.



∴

Solution :-

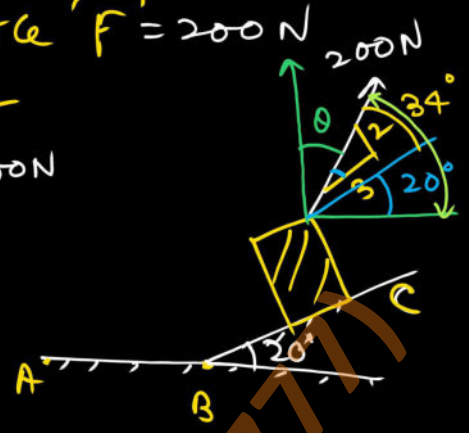
Component of force 'F' = 200 N perpendicular to 'AB' :-

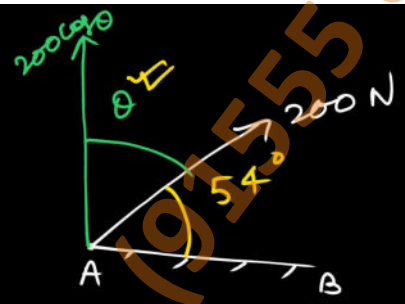


$\tan\alpha = \frac{2}{3}$

$\therefore \alpha = \tan^{-1}\left(\frac{2}{3}\right)$

$\approx 33.69^\circ \approx 34^\circ$





$\theta = 90^\circ - 54^\circ$

$= 36^\circ$

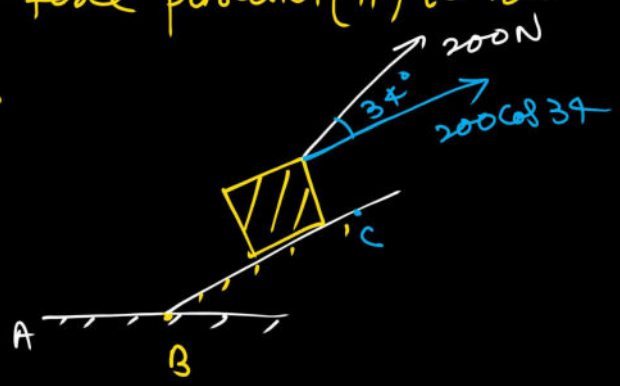
$\therefore$ Component of force perpendicular to 'AB'

$= 200\cos\theta$

$= 200 \times \cos 36$

$= 161.80$ N Ans//

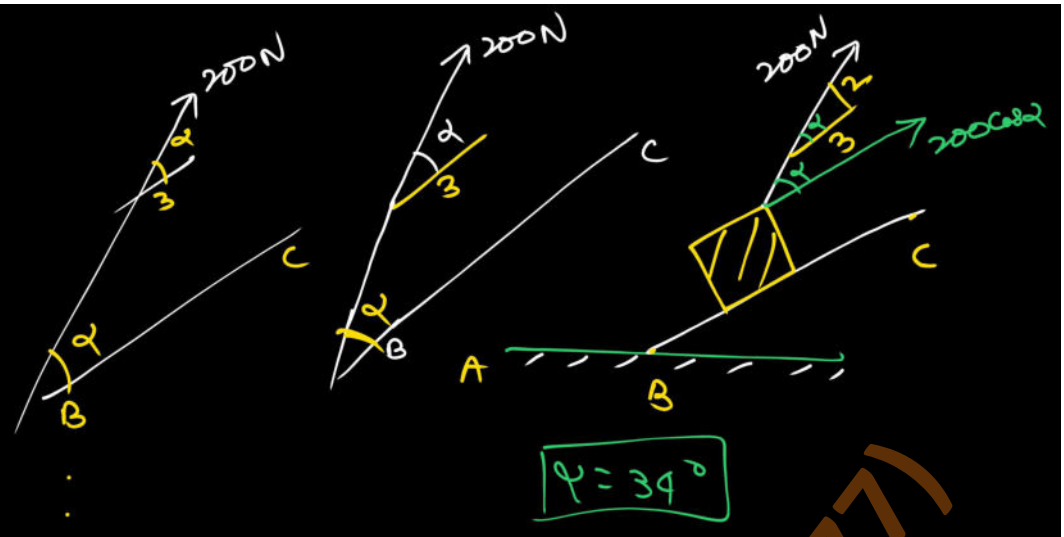
→ Component of force parallel (||) to 'BC'



$= 200\cos 34^\circ$

$= 165.80$

Ans//



Algebra of vectors :→

→ Summation (addition), substraction and multiplication of vectors are govern by seperate rules which are called algebra of vectors.

Multiplication of vectors :→

→ There are two types of multiplication of vectors :→

I Dot (·) product of vectors.

II Cross (×) product of vectors.

I Dot (·) product :→

→ Dot product of two vectors $\vec{A}$ and $\vec{B}$ is given by :—

$$\boxed{\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta}$$

where θ = Angle between $\vec{A}$ and $\vec{B}$.

** → Dot product of two vector is always a Scalar.

** → Dot product follow commutative law, and distributive law.

** $\boxed{\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}}$ → commutative law.

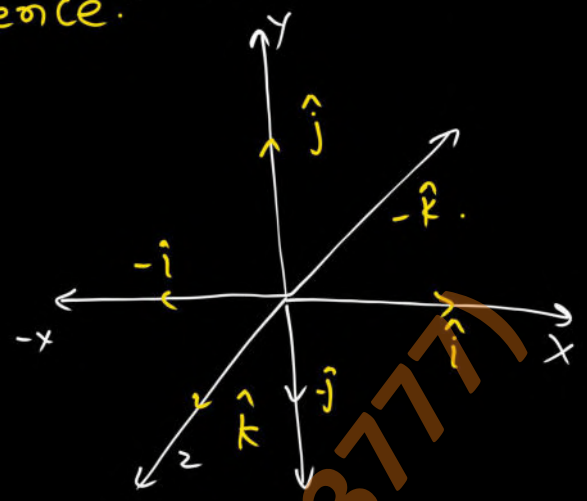
* $\boxed{\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}}$ → Distributive law.

** → If Dot product of two vectors $(\vec{A}\cdot\vec{B})$ is zero(0), then vectors are called Solenoidal vector.

if $\boxed{\vec{A}\cdot\vec{B} = 0}$

Then $\vec{A}$ and $\vec{B}$ are called "Solenoidal vector"

→ physical meaning of dot product is convergence.



$\hat{i}\cdot\hat{j} = |\hat{i}||\hat{j}|\cos 90^\circ$

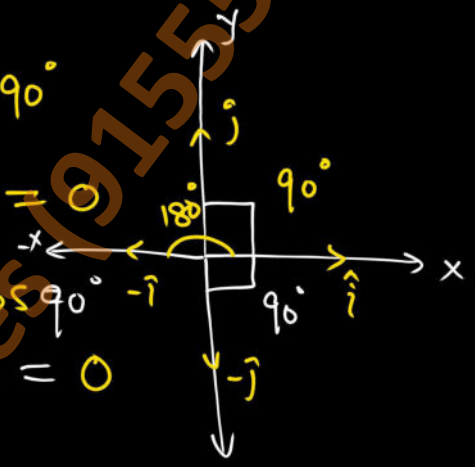
$= 1\times 1\times 0 = 0$

$\hat{i}\cdot(-\hat{j}) = |\hat{i}||-\hat{j}|\cos 90^\circ$

$= 1\times 1\times 0 = 0$

$\hat{i}\cdot\hat{i} = |\hat{i}||\hat{i}|\cos 0$

$= 1\times 1\times 1 = 1$

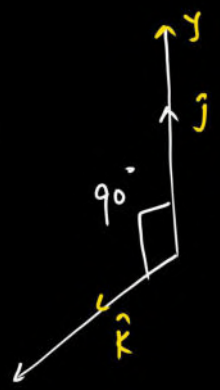


$\hat{j}\cdot\hat{k} = 0$

$$\boxed{\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0}$$

$\hat{i}\cdot(-\hat{i}) = |\hat{i}||-\hat{i}|\cos 180^\circ$

$= 1\times 1\times(-1) = -1$



$$\Rightarrow \boxed{\hat{i}\cdot\hat{i} = -\hat{i}\cdot-\hat{i} = \hat{j}\cdot\hat{j} = -\hat{j}\cdot-\hat{j} = \hat{k}\cdot\hat{k} = -\hat{k}\cdot-\hat{k} = 1}$$

$$\boxed{\hat{i}\cdot(-\hat{i}) = \hat{j}\cdot(-\hat{j}) = \hat{k}\cdot(-\hat{k}) = -1}$$

Cross product of vectors :—

→ Cross product of two vectors $\vec{A}$ and $\vec{B}$ is given by

$$\boxed{\vec{A}\times\vec{B} = |\vec{A}||\vec{B}|\sin\theta\,\hat{n}}$$

$|\vec{A}|$ = magnitude of $\vec{A}$.
$|\vec{B}|$ = magnitude " $\vec{B}$.

θ = Angle between $\vec{A}$ and $\vec{B}$.

$\hat{n}$ = unit vector gives direction of cross (X) product of two vectors $\vec{A}$ and $\vec{B}$.

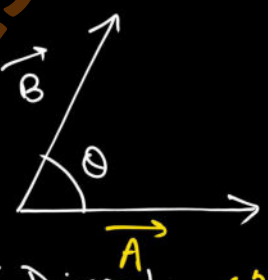
= This unit vector will lie perpendicular to the plane on which $\vec{A}$ and $\vec{B}$ lie.

→ To find direction $\hat{n}$, Right hand thumb is used:—

Right hand thumb Rule :→

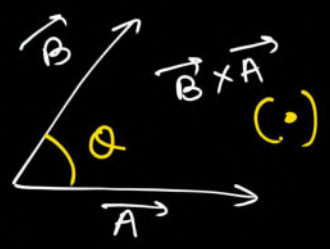
→ If we have to find direction of $\vec{A}\times\vec{B}$ then put palm of right hand along $\vec{A}$ and then rotate the fingers through angle between $\vec{A}$ and $\vec{B}$, then in this condition thumb of right will give direction($\hat{n}$) of $\vec{A}\times\vec{B}$.

→ let $\vec{A}$ and $\vec{B}$ lie on the plane paper.



$\vec{A}\times\vec{B}$ = Direction($\hat{n}$) of $\vec{A}\times\vec{B}$ is perpendicular to plane of paper and outward (X).

$\vec{B}\times\vec{A}$ = Direction($\hat{n}$) will be perpendicular to plane and inward (•)



$\vec{B}\times\vec{A}$ (•)

$\vec{A}\times\vec{B}$ (X)

→ Cross product of two vectors is always a vector.

→ Cross product of two vectors does not follow commutative law.

$$\boxed{\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}}$$

$$|\vec{A} \times \vec{B}| = |\vec{B} \times \vec{A}|$$

→ If cross product of two vectors $(\vec{A}, \vec{B})$ is zero, then vectors are called irrotational vectors

if $\boxed{\vec{A} \times \vec{B} = 0}$

then $\vec{A}$ and $\vec{B}$ are called irrotational vector.

→ Physical meaning of cross product is curl.

Engg. Mechanics

V.V.I ****

MOMENT ($\vec{M}$) :→

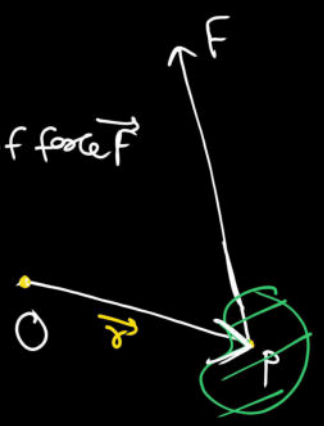
→ The turning effect of a force is called moment.

→ moment due a force (F) about a point (O) is given by,

moment about point 'O'

$$\boxed{\vec{M}_O = \vec{r} \times \vec{F}}$$

where $\vec{r}$ = position vector of force $\vec{F}$ from point 'O' about which moment is required.



→ Moment is a vector quantity.

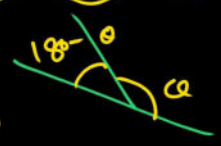
→ SI unit of moment is Newtonmeter (Nm)

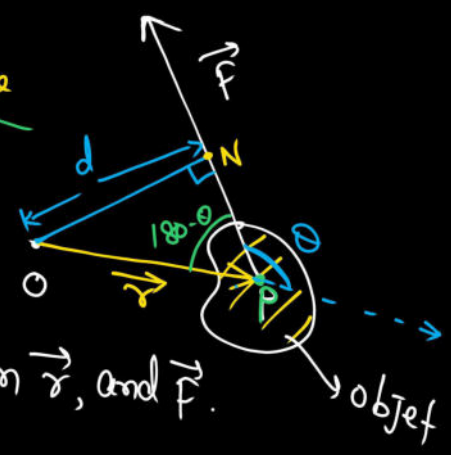
Derivation for moment :→

∵ We know that

$$\vec{M}_O = \vec{r} \times \vec{F}$$

$$= |\vec{r}||\vec{F}|\sin\theta\,\hat{n}$$





where θ = angle between $\vec{r}$, and $\vec{F}$.

$$|\vec{M_o}| = M_o = |\vec{r}||\vec{F}| \sin\theta$$

$$\boxed{M_o = r F \sin\theta} \rightarrow (1)$$

where

$M_o \rightarrow$ magnitude of moment

$r \rightarrow$ Magnitude of position vector

$F \rightarrow$ Magnitude of force vector.

Again In Right angle triang ONP,

$$\sin(180-\theta) = \frac{d}{r}$$

$$\Rightarrow \boxed{d = r \sin\theta}$$

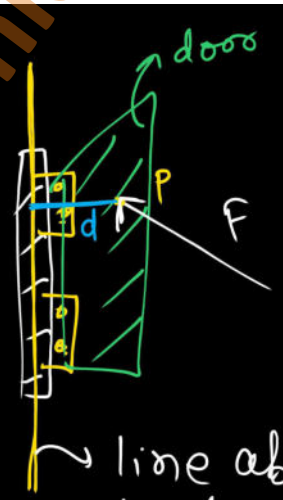
putting value of "$r\sin\theta = d$" in eqn (1)

we get

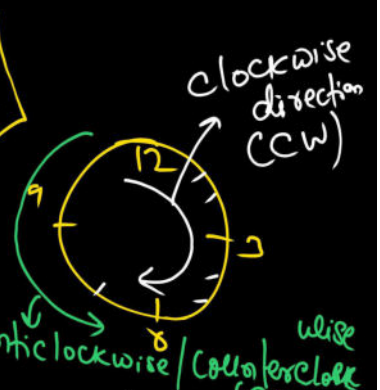
$$\because \quad M_o = r F \sin\theta$$

$$\boxed{M_o = F d}$$

where d = perpendicular distance of line of action of force (F) from point 'O' about which moment is required.



$$\boxed{M = F d}$$



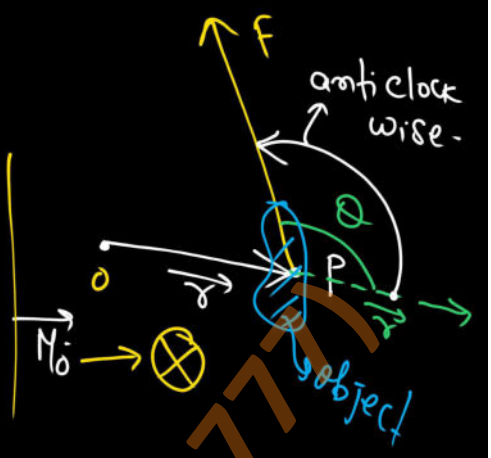
$\therefore$ line about which moment (M) is produced by force (F)

Direction of moment ($\vec{M_o}$) :→

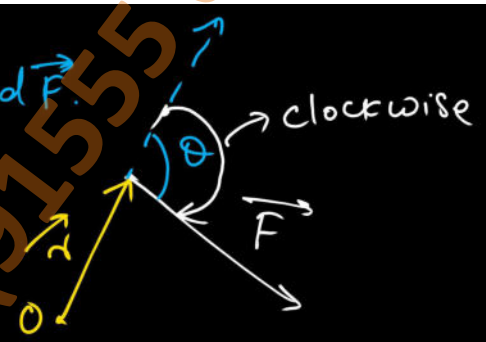
$\vec{M_o} = \vec{r} \times \vec{F}$

θ = angle between $\vec{r}$ and $\vec{F}$.

direction of moment → anticlockwise



eg. θ = angle b/w $\vec{r}$ and $\vec{F}$.



Moment (M) about point 'O' due to force $\vec{F}$ = clockwise.

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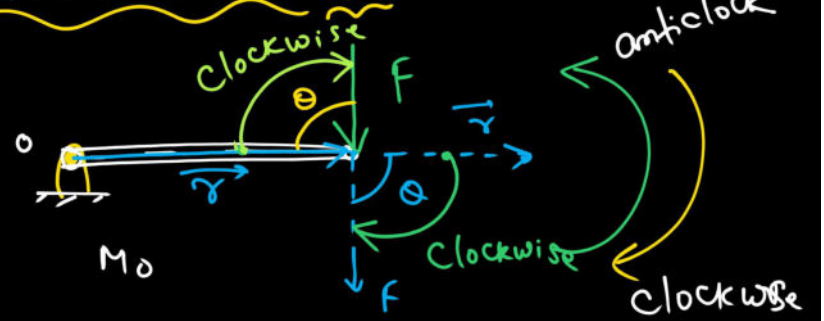
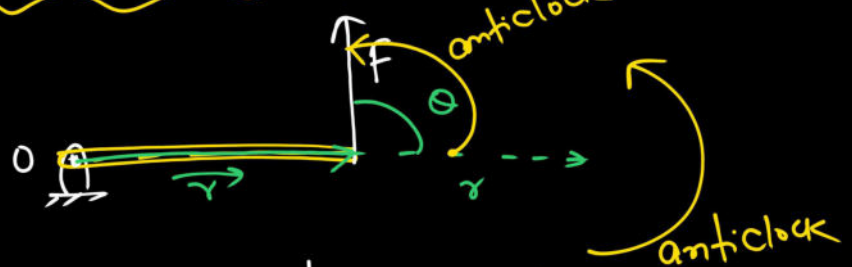
Clockwise moment (M) :→

$\vec{M_o} = \vec{r} \times \vec{F}$



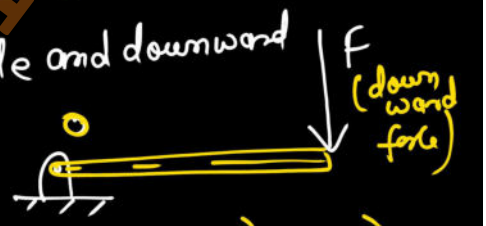
Anticlockwise moment :→



M_O → Direction → anticlock moment.

points to remember for Clockwise and anticlockwise moment :→

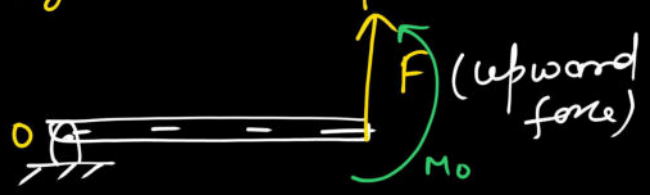
(a) If force is right side and downward of point (O) about which moment is required then moment is Clockwise



Moment about 'O' (M_O)

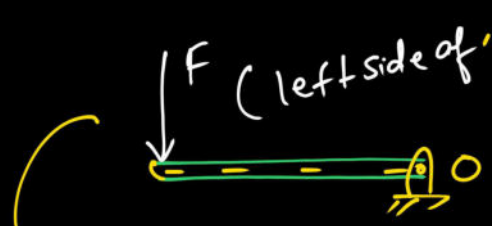
Clockwise

(b) If force is right side and upward from point 'O'



then moment (M_O) → Anticlockwise.

(c)

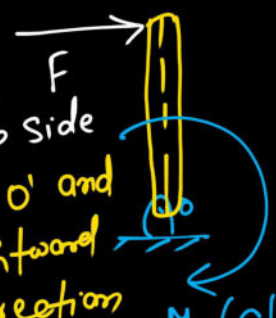


M_O (anticlockwise)

(d)

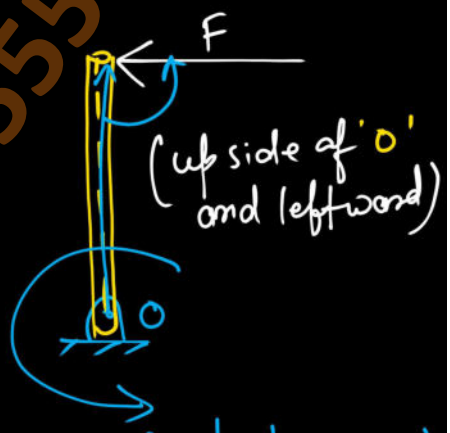
$M_O \rightarrow$ clockwise.

(i)



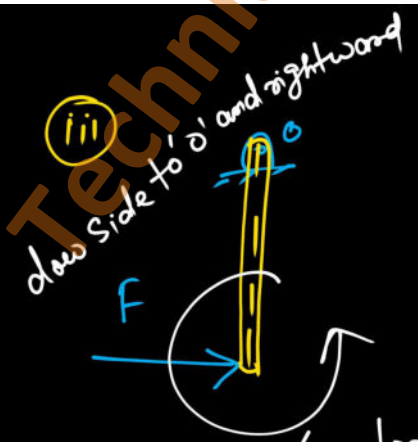
M_O (Clockwise)

(ii)



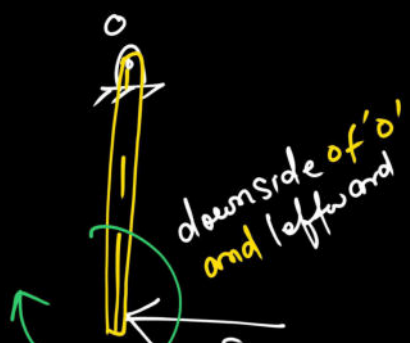
M_O (anticlockwise)

(iii)



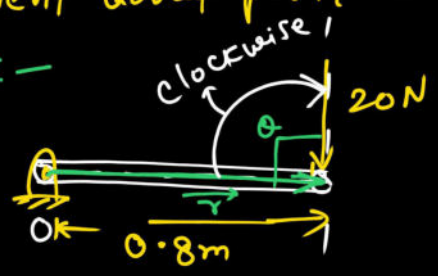
M_O (anticlockwise)

(iv)



M_O (Clockwise).

Q. Find moment about point 'O' for given fig. below:-



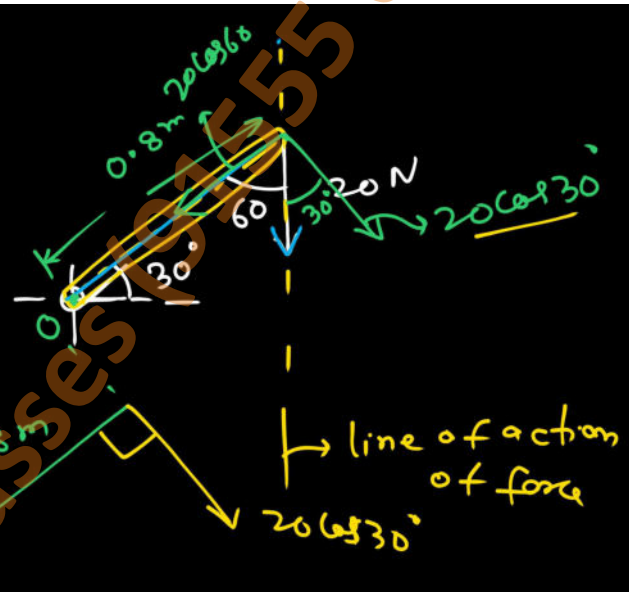
$M_O = Fd$

$= 20N \times 0.8m$

$= 16\ Nm$ clockwise

line of action force.

Q. $M_O = ?$



$M_O =$

$M_O = 20\cos(30) \times 0.8\ N\text{-}m + 20\cos 60 \times 0$

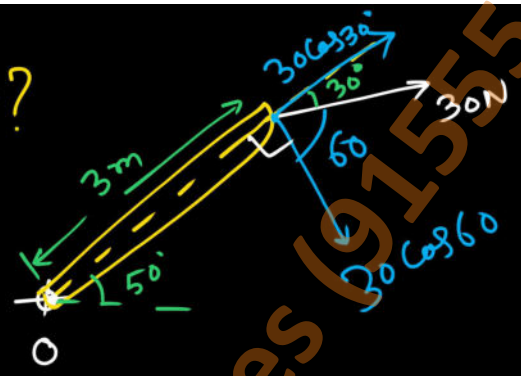
$= 20 \times \cos 30 \times 0.8$

$= 13.85\ N\text{-}m$ Ans// clockwise.

Sign Convention :→

→ In generally clockwise moment is taken as positive moment while anticlockwise moment is taken as negative moment.

Q $M_O = ?$

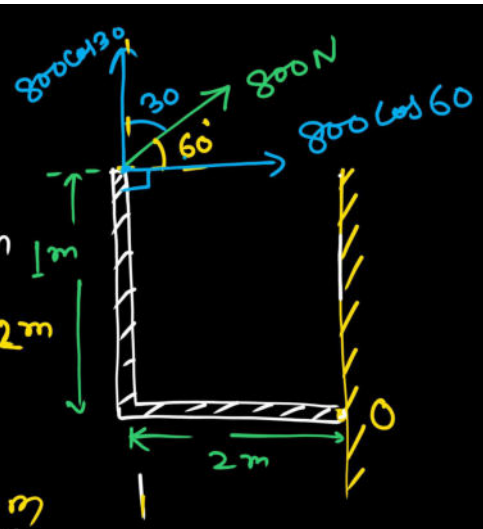


$M_O = +30\cos 60 \times 3m + 30\cos 30 \times 0$

$= 30\cos 60 \times 3m + 0 = 45 N\text{-}m$

Ans//

Q $M_O = ?$

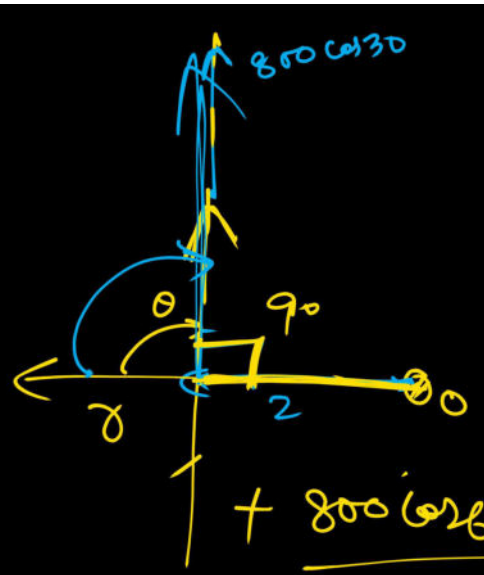


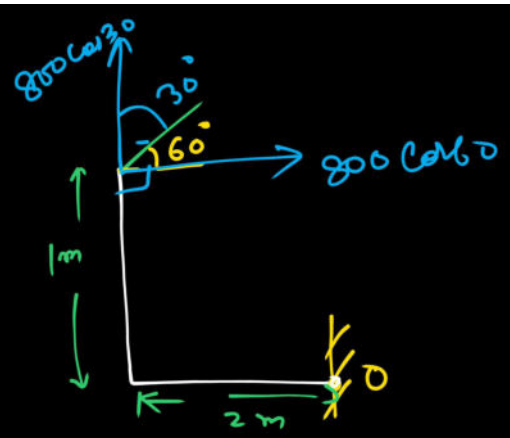
$M_O = +800\cos 60 \times 1m$

$+ 800\cos 30 \times 2m$

$= 1785.64$ N-m

Ans//





$+ 800\cos 60 \times 1m + 800\cos 30 \times 2m$

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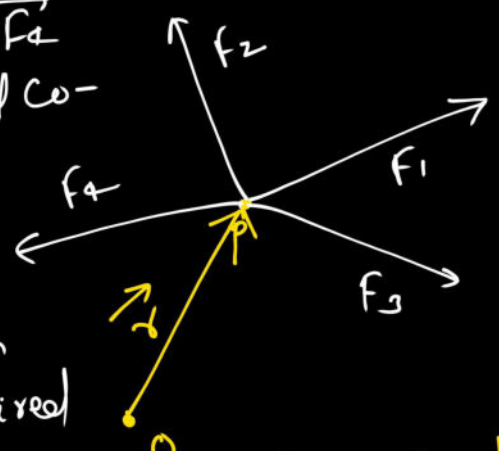
Varignon's principle :→

→ This principle states that If there is coplanar and concurrent system of forces, then moment due to all forces about a point will be equal to moment due to resultant force of all the forces about same point.

let $\vec{F_1}$, $\vec{F_2}$, $\vec{F_3}$ and $\vec{F_4}$ are concurrent and co-plamar forces.



let 'O' is the point about which moment is required

let $\vec{r}$ = Position Vector of all the forces from point 'O'

$$\vec{M}_O = \vec{r}\times\vec{F_1} + \vec{r}\times\vec{F_2} + \vec{r}\times\vec{F_3} + \vec{r}\times\vec{F_4}$$

$$= \vec{r}\times(\vec{F_1} + \vec{F_2} + \vec{F_3} + \vec{F_4})$$

$$\boxed{= \vec{r}\times\vec{F_R} = \vec{M}_O}$$

$\vec{M}_O$ ↓ moment about point O.

Where $\vec{F_R}$ = Resultant force of all the forces $\vec{F_1}, \vec{F_2}, \vec{F_3}, \vec{F_4}$.

V.V.I. Composition of forces :→

→ Composition of forces is the reverse process of resolution of forces.

→ The process of vectorally adding number of forces and to form a equivalent force is called composition forces.

→ Resultant force (F_R) :→

A single equivalent forces of number of forces by composition is called resultant force.

There following method of composition of forces or to find Resultant force:→

Method of find Resultant force

- Analytical method
  - law of parallelogram (Trigonometrical Method)
  - Resolution Method.
- Geometrical Method.
  - Triangle law.
  - polygon law.

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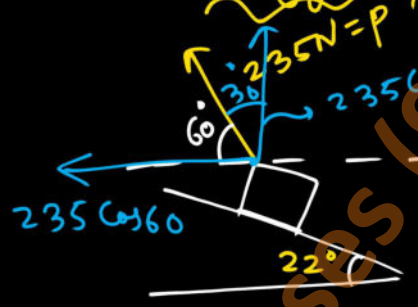
Q- 1

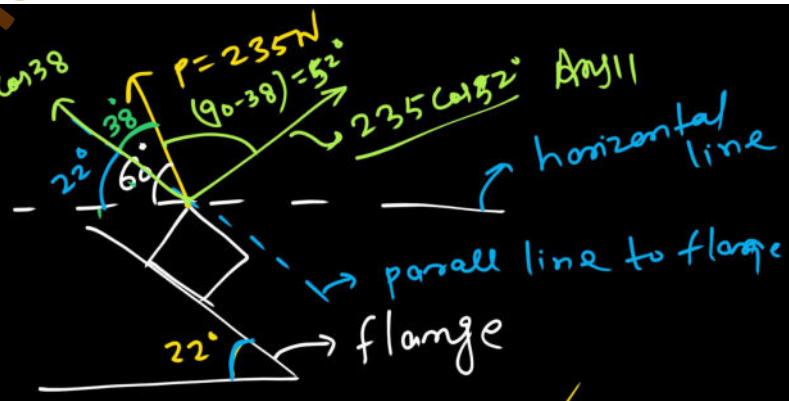
Q

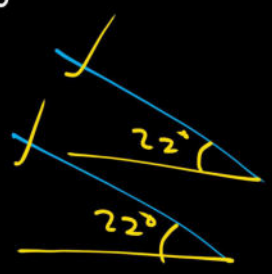


⇒ vertical component of force 'P' = 235 cos30

⇒ and horizontal component of force 'P' = 235 cos60 N

Q-2

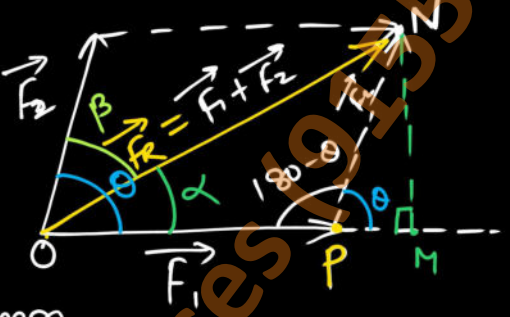




Law of parallelogram (Trigonometrical law) :→

→ This law states that if two forces are taken as two adjacent side of a parallelogram, then a diagonal of parallelogram represents resultant force of that two forces.

let $\vec{F_1}$ and $\vec{F_2}$ are two forces and 'θ' is the angle between them



$\vec{F_R}$ = Resultant force of $\vec{F_1}$ and $\vec{F_2}$

i.e. $\boxed{\vec{F_R} = \vec{F_1} + \vec{F_2}}$

let $|\vec{F_1}| = F_1$ = magnitude of force $\vec{F_1}$.

$|\vec{F_2}| = F_2$ = " " " $\vec{F_2}$

$|\vec{F_R}| = F_R$ = " " " $\vec{F_R}$

let Draw a perpendicular on line of action of force $\vec{F_1}$ from point 'N'

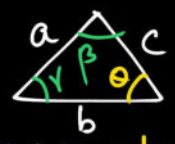
In Triang 'ONP'

By cosine law:→

$$\cos(180-\theta) = \frac{F_1^2+F_2^2-F_R^2}{2F_1\times F_2}$$

$$\Rightarrow \quad \frac{-\cos\theta}{1} = \frac{F_1^2+F_2^2-F_R^2}{2F_1F_2}$$

Cosine law



By cosine law

$$\cos\theta = \frac{b^2+c^2-a^2}{2bc}$$

$$\cos\beta = \frac{a^2+c^2-b^2}{2ac}$$

$$\cos\gamma = \frac{a^2+b^2-c^2}{2ab}$$

$$\Rightarrow \quad -2F_1F_2\cos\theta = F_1^2+F_2^2-F_R^2$$

$$\Rightarrow \quad F_R^2 = F_1^2+F_2^2+2F_1F_2\cos\theta$$

$$\Rightarrow \quad \boxed{F_R = \sqrt{F_1^2+F_2^2+2F_1F_2\cos\theta}}$$

↳ Magnitude of Resultant force ($\vec{F_R}$)

→ Direction of Resultant force ($\vec{F_R}$) :—

let α = angle between $\vec{F_1}$ and $\vec{F_R}$.

and β = " " $\vec{F_2}$ and " .

In Right angle triangle △'OMN'

$$\tan\alpha = \frac{NM}{OM}$$

$$\boxed{\tan\alpha = \frac{NM}{OP+PM}} \quad (1)$$

Again in Right angle triangle 'PMN'

$$\sin\theta = \frac{MN}{F_2} \Rightarrow \boxed{MN = F_2 \sin\theta}$$

Again in $\triangle$ PMN

$$\cos\theta = \frac{PM}{F_2} \Rightarrow \boxed{PM = F_2 \cos\theta}$$

$\therefore$ from equation ① we can write

$$\boxed{\tan\alpha = \frac{MN}{OP + PM} = \frac{F_2 \sin\theta}{F_1 + F_2 \cos\theta}}$$

$$\boxed{\tan\beta = \frac{F_1 \sin\theta}{F_2 + F_1 \cos\theta}}$$

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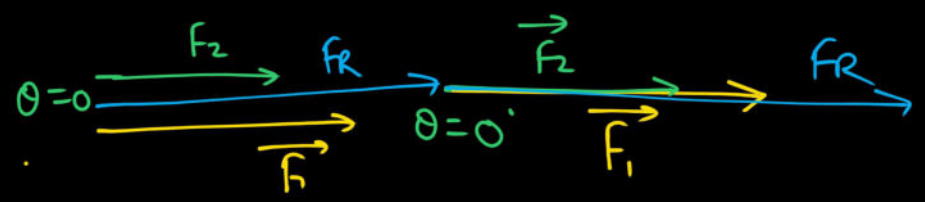
Some special cases related to law of parallelogram :→

→ Case – I, let $\theta = 0^\circ$ between $\vec{F_1}$ and $\vec{F_2}$.



∴ Resultant force (F_R) :—

$$F_R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$= \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos 0}$$

$$= \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \times 1}$$

$$= \sqrt{(F_1 + F_2)^2}$$

$$\Rightarrow \boxed{F_R = F_1 + F_2}$$

F_1 → +ve, F_2 → +ve

→ Maximum value of resultant force of two force F_1 and F_2

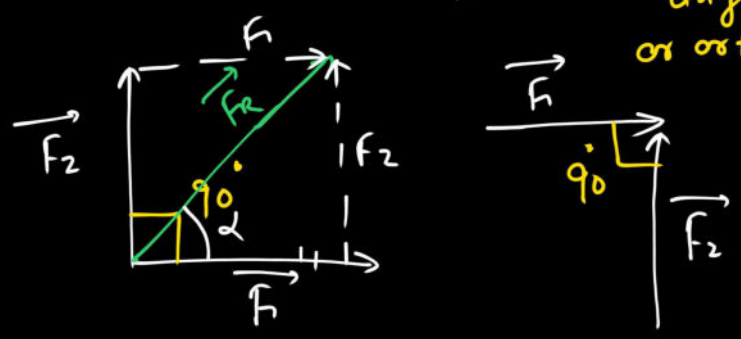
→ Direction of $\vec{F_R}$:—

$$\tan\alpha = \frac{F_2\sin\theta}{F_1 + F_2\cos\theta} = \frac{F_2\sin 0}{F_1 + F_2\cos 0} \quad (\sin 0 \to 0)$$

$$\tan\alpha = 0$$

$$\boxed{\alpha = 0^\circ}$$

Case-II when $\theta = 90^\circ$ (orthogonal angle or orthogal forces)



∴

$$\therefore \quad F_R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$= \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos 90}$$

$$= \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \times 0}$$

$$** \quad \boxed{F_R = \sqrt{F_1^2 + F_2^2}}$$

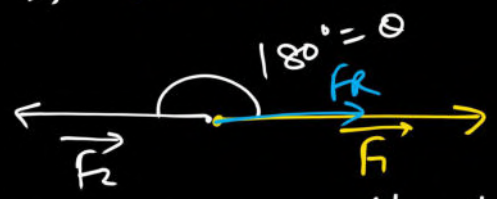
→ Direction of Resultant force (F_R) :→

$\alpha \rightarrow \vec{F_R}$ makes $\vec{F_1}$.

$$\tan\alpha = \frac{F_2 \sin\theta}{F_1 + F_2\cos\theta} = \frac{F_2 \sin 90}{F_1 + F_2 \cancel{\cos 90^\circ}^{\,0}}$$

$$\therefore \quad \boxed{\tan\alpha = \frac{F_2}{F_1}}$$

Case – III If $\theta = 180^\circ$



Two forces are opposite in direction

$\therefore$ Magnitude of Resultant force (F_R):—

$$\because F_R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$= \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos 180}$$

$$= \sqrt{F_1^2 + F_2^2 + 2F_1F_2(-1)}$$

$$= \sqrt{F_1^2 + F_2^2 - 2F_1F_2}$$

$$\therefore F_R = \sqrt{(F_1 - F_2)^2}$$

$$\boxed{F_R = |F_1 - F_2|} \rightarrow$$ Minimum value of Resultant of two force $\vec{F_1}$ and $\vec{F_2}$.

→ Direction of Resultant force (F_R) :—

$$\tan\alpha = \frac{F_2 \sin\theta}{F_1 + F_2\cos\theta}$$

$$= \frac{F_2 \sin 180}{F_1 + F_2\cos 180} = 0$$

$$\tan\alpha = 0$$

$$\boxed{\alpha = 0^\circ}$$

Note :→

(a) Magnitude of Resultant force (F_R) lies between its maximum and minimum value.

$$\text{Min}^m \le F_R \le \text{Max}^m$$

$$\boxed{|F_1 - F_2| \le F_R \le F_1 + F_2}$$

$\downarrow \theta = 180^\circ$ $\qquad$ $\downarrow \theta = 0^\circ$

Engg. Mechanics

Q Two orthogonal forces of equal magnitude 'P', then their resultant will be

(a) $\frac{P}{2}$ (b) $2P$ (c) $\sqrt{2}P$ (d) $\frac{P}{\sqrt{2}}$.

Solⁿ: $\theta = 90^\circ$, $F_1 = F_2 = P$

$$\therefore F_R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos 90^\circ} = \sqrt{P^2 + P^2} = \sqrt{2}P$$

Q. The maximum and minimum value of resultant force of two forces are 12 N and 8N, then two forces are

(a) 10N, 4N (b) 8N, 4N (c) 10N, 8N

(d) 10 N, 2N.

Sol^n:- let bigger force (F_1)
smaller force (F_2)

Given max^m = $F_1 + F_2 = 12N$

min^m = $F_1 - F_2 = 8N$

$2F_1 = 20N$

$F_1 = 10N$

$\therefore F_2 = 2N$ Ans

Q. Two forces of equal magnitude 'a' and 'θ' is the angle between them their resultant force will be

(a) $2a\cos\theta$ (b) $2a\sin\theta$ (c) $2a\cos\frac{\theta}{2}$ (d) $2a\sin\frac{\theta}{2}$

Sol^n:- Given, $F_1 = F_2 = a$
$\theta \rightarrow$ angle.

$F_R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$

$$F_R = \sqrt{a^2 + a^2 + 2a \times a \cos\theta}$$

$$= \sqrt{a^2 + a^2 + 2a^2 \cos\theta}$$

$$= \sqrt{2a^2 + 2a^2 \cos\theta}$$

$$= \sqrt{2a^2 (1 + \cos\theta)}$$

$$= \sqrt{2a^2 \times 2\cos^2 \frac{\theta}{2}}$$

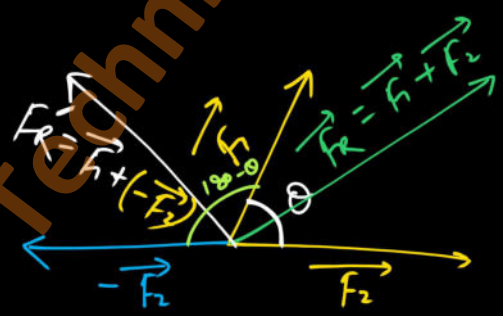
$$\boxed{F_R = 2a\cos\frac{\theta}{2}}$$

$\because$ $1 + \cos\theta = 2\cos^2\frac{\theta}{2}$

$1 - \cos\theta = 2\sin^2\frac{\theta}{2}$

Q Two forces of equal magnitude 'a' and 'θ' is the angle between them, then the magnitude of resultant force of their substraction.

(a) $2a\sin\theta$ (b) $2a\cos\theta$ (c) $2a\cos\frac{\theta}{2}$ (d) $2a\sin\frac{\theta}{2}$.



$|\vec{F_1}| = |\vec{F_2}| = a$

$\vec{F_1} - \vec{F_2}$ or $\vec{F_2} - \vec{F_1}$

$= \vec{F_1} + (-\vec{F_2})$

$\therefore$ Resultant force (F_R') of subraction/difference between two $\vec{F_1}$ and $\vec{F_2}$

$$\therefore \vec{F_R'} = \vec{F_1} + (-\vec{F_2})$$

$$F_R' = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos(180-\theta)}$$

$$= \sqrt{F_1^2 + F_2^2 + 2F_1F_2(-\cos\theta)}$$

$$\boxed{F_R' = \sqrt{F_1^2 + F_2^2 - 2F_1F_2\cos\theta}}$$

$\therefore$ if $F_1 = F_2 = a$

$$F_R' = \sqrt{a^2 + a^2 - 2 \times a \times a\cos\theta}$$

$$= \sqrt{2a^2 - 2a^2\cos\theta}$$

$$= \sqrt{2a^2(1-\cos\theta)}$$

$$= \sqrt{2a^2 \times 2\sin^2\frac{\theta}{2}}$$

$$\boxed{F_R' = 2a\sin\frac{\theta}{2}}$$ Ans//

Q If the angle between two non zero forces $\vec{F_1}$ and $\vec{F_2}$ be 120° and its resultant is 'F' then

(a) 'F' must be equal to $|F_1 - F_2|$

(b) F must be less than $|F_1 - F_2|$

(c) F must be greater than $|F_1 - F_2|$

(d) F may be equal to $|F_1 - F_2|$

Q. Two forces act at an angle of 120°. If the greater force is 50N and their resultant is perpendicular to smaller force, then magnitude of small will be ?

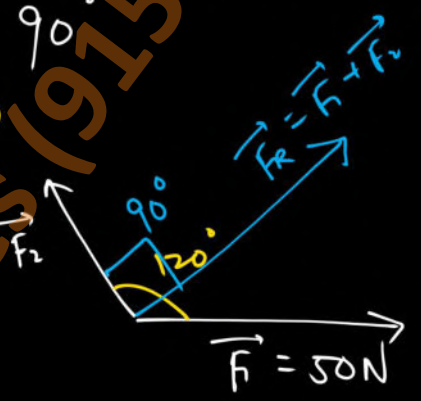
Solution :→ Given

Angle between two force (θ) = 120°

let bigger force is 'F_1' magnitude = 50N

Angle between resultant (F_R) and smaller force (F_2) = 90°

then F_2 = ?



We know, that

$$\tan 90 = \frac{F_1 \sin\theta}{F_2 + F_1 \cos\theta}$$

$$\infty = \frac{50N \sin 120°}{F_2 + 50 \cos 120°}$$

$$\Rightarrow \frac{1}{0} = \frac{50 \sin 120}{F_2 + 50 \cos 120}$$

$$\Rightarrow F_2 + 50\cos 120 = 0$$

$$\Rightarrow F_2 = -50\cos 120$$

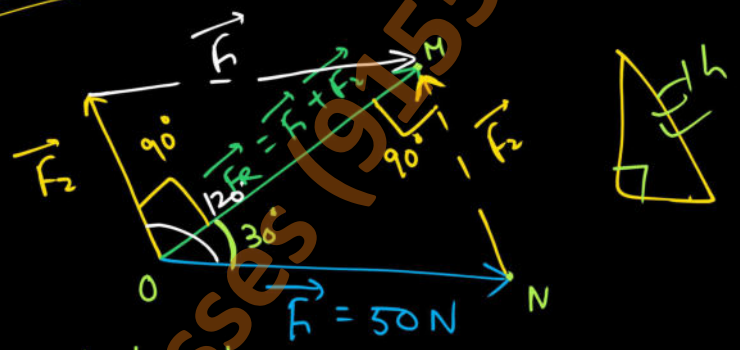
$$= -50\cos(90+30)$$

$$= -50(-\sin 30)$$

$$= +\cancel{50} \times \frac{1}{\cancel{2}} = 25N$$

$$\boxed{F_2 = 25N \text{ Ans//}}$$

2nd Method



Right angle triangle

OMN, $\sin 30 = \frac{F_2}{50}$,

$$\Rightarrow 50\sin 30 = F_2$$

$$\Rightarrow F_2 = 50 \times \frac{1}{2} = 25N \text{ Ans//}$$

Again in $\triangle$ OMN

$$\cos 30 = \frac{F_R}{50N}$$

$$F_R = 50\cos 30 = 50 \times \frac{\sqrt{3}}{2} = 25\times\sqrt{3}\,N$$

$$= 43.30N \text{ Ans//}$$

Q The resultant of forces 'P' and 'Q' is of magnitude 'Q' Show that when the force 'Q' is doubled and 'P' remaining same, the new resultant is right angle to 'P' and its magnitude will be $\sqrt{4Q^2 - P^2}$

Solⁿ:— Two forces 'P' and 'Q'

Resultant (FR) = Q

let angle = θ = between P and Q

$\therefore F_R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$

$Q = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

$\Rightarrow \cancel{Q^2} = P^2 + \cancel{Q^2} + 2PQ\cos\theta$

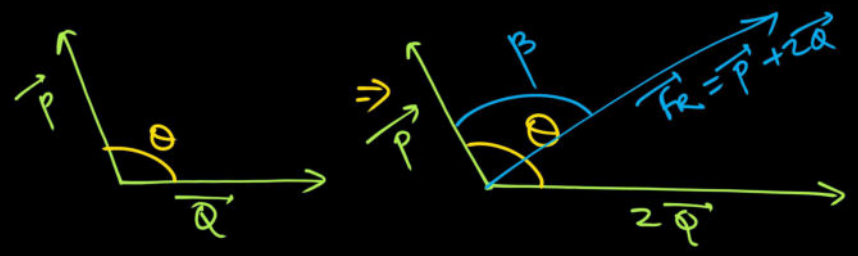
$\Rightarrow P^2 + 2PQ\cos\theta = 0$ —— (II)

$\Rightarrow P(P + 2Q\cos\theta) = 0$ —— (I)

According to question 'Q' is doubled means

2Q

P ⟶ same



let β = angle b/w $\vec{P}$ and Resultant

$$\therefore \tan\beta = \frac{2Q\sin\theta}{P + 2Q\cos\theta}$$

from eqn ①

$$\tan\beta = \frac{2Q\sin\theta}{0} \qquad \left[\because \text{eqn ①}\ P + 2Q\cos\theta = 0\right]$$

$$= \infty$$

$$\boxed{\beta = 90^\circ}$$ proved//

Again Resultant $F_R = \sqrt{(P)^2 + (2Q)^2 + 2P \times 2Q\cos\theta}$

$$= \sqrt{P^2 + 4Q^2 + 4PQ\cos\theta}$$

$$= \sqrt{P^2 + 2PQ\cos\theta + 2PQ\cos\theta + 4Q^2}$$

$$= \sqrt{0 + 2PQ\cos\theta + 4Q^2} \quad [\text{from eqn ①}]$$

$F_R = \sqrt{4P^2 - P^2}$ proved//

Engg. Mechanics

Resolution method :→

→ Limitations of law of parallelogram is that resultant force of only two forces can be find out.

→ But By resolution method, we can find out resultant force of two or more than two forces.

→ followings steps are involve in resolution method :→

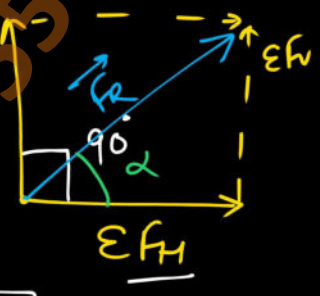
Step-I :→ Resolve all the forces in horizontal direction and find algebraic summation of all the horizontal component that is ΣF_H.

ΣF_H → force; → Algebraic summation of all horizontal forces. → Horizontal direction; → Summation

Step-II: → Resolve All the forces in vertical direction and find Algebraic Summation of all the vertical components. that is ΣF_V.

where ΣF_V → Algebraic summation of vertical Components.

Step-III: - Then find Resultant force of all the forces by the formula:—



**

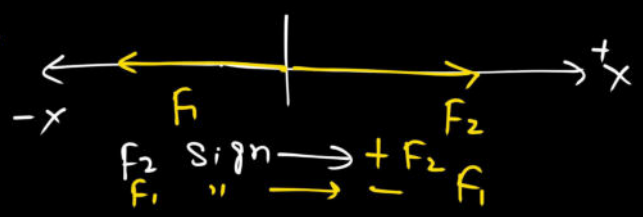
$$F_R = \sqrt{(\Sigma F_V)^2 + (\Sigma F_H)^2}$$

Direction of resultant force (F_R): →

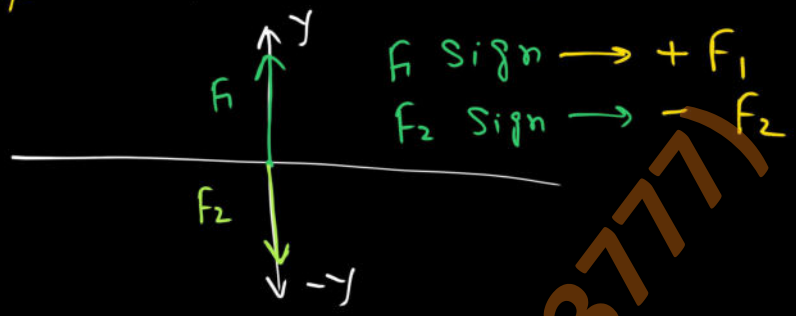
$$\tan\alpha = \frac{\Sigma F_V}{\Sigma F_H}$$

Sign convention: →

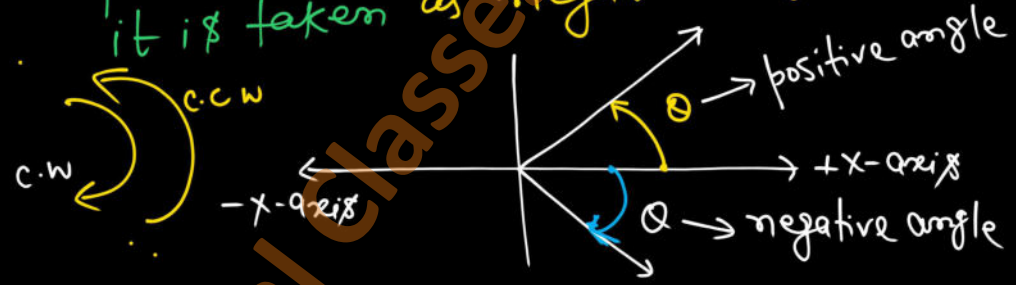
(i) Horizontal forces acting towards positive x-axis are taken with positive sign while horizontal forces acting towards negative x-axis are taken with negative sign.



(ii) All the forces acting along positive y-axis are taken with positive sign while all the vertical forces acting along −ve y-axis are taken with Negative sign.



(iii) If angle(θ) is measured from +ve x-axis in anticlockwise then it is taken as positive angle(θ) but if it is measured from + x-axis in clockwise direction it is taken as negative angle(θ).



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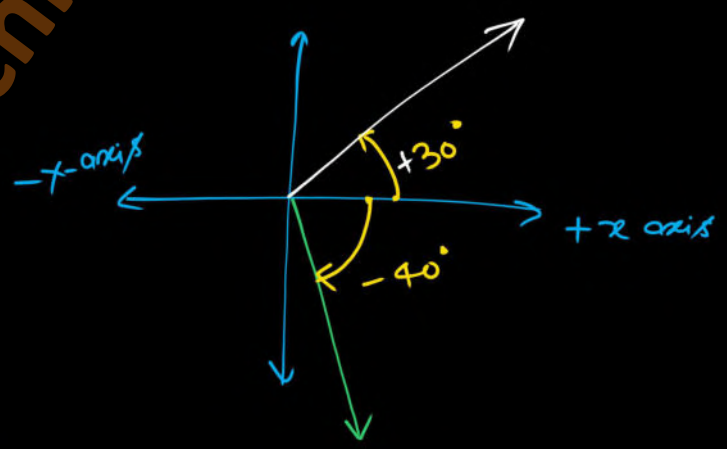
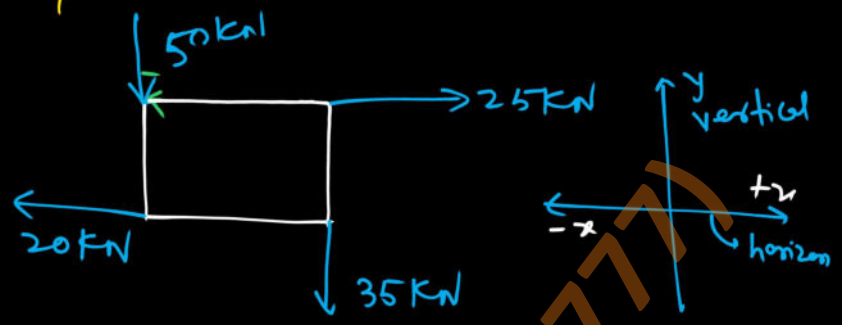


Engg. Mechanics

Q Find Resultant force of following given system of forces.



$$\Sigma F_H = +25KN - 20KN = +5KN$$

$$\Sigma F_V = -35KN - 50KN = -85KN$$

∴ Magnitude of Resultant force (F_R) :→

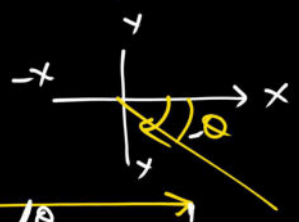
$$F_R = \sqrt{(\Sigma F_H)^2 + (\Sigma F_V)^2}$$

$$= \sqrt{(5KN)^2 + (-85KN)^2}$$

$$F_R = \sqrt{25 + 7225}\ KN$$

$$= 85.147\ KN \quad \text{Ans//}$$

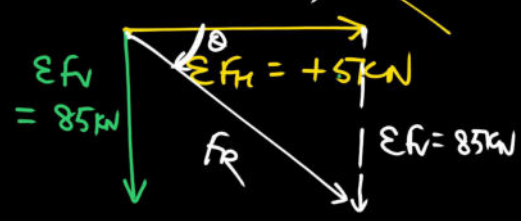
Direction of F_R :→



$$\tan\theta = \frac{\Sigma F_V}{\Sigma F_H} = \frac{85KN}{5KN}$$

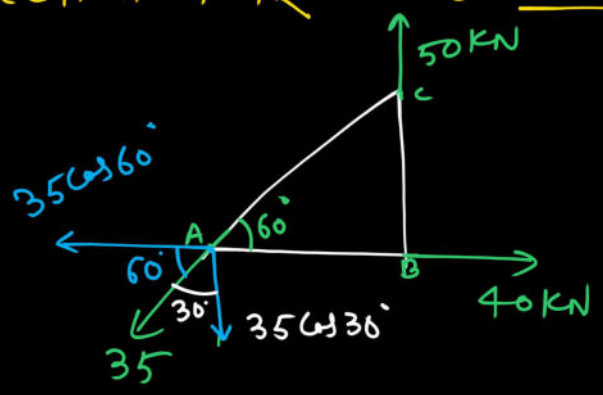
$$\theta = \tan^{-1}(17)$$

$$= 86.63^\circ$$



$\therefore$ Direction of $F_R = -86.63^\circ$ from +x-axis

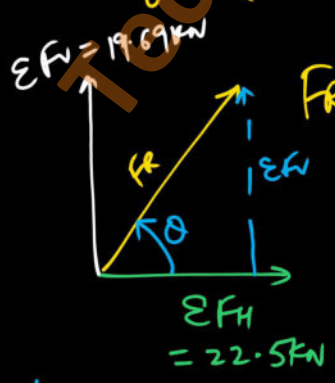
Q



$$\therefore \Sigma F_H = +40\,KN - 35\cos 60$$
$$= 40\,KN - 35 \times \frac{1}{2}$$
$$= 40\,KN - 17.5$$
$$= 22.5\,KN$$

Again $\Sigma F_V = +50\,KN - 35\cos 30$
$$= 50\,KN - 35 \times \frac{\sqrt{3}}{2}$$
$$= 19.69\,KN$$

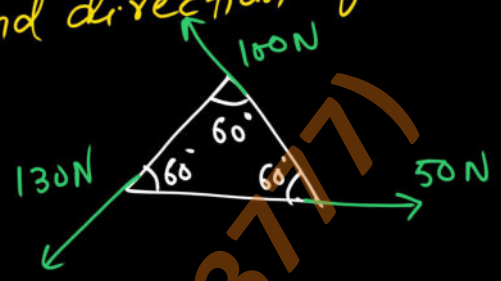
Mag. Resultant force (F_R) of all the forces is



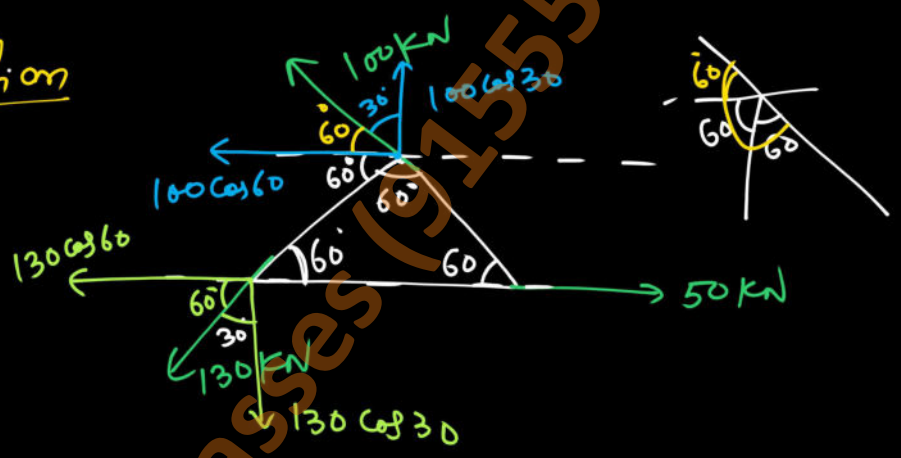
$$F_R = \sqrt{(\Sigma F_H)^2 + (\Sigma F_V)^2}$$
$$= \sqrt{(22.5\,KN)^2 + (19.69)^2}$$
$$= 29.89\,KN$$
$$\approx 30\,KN \quad \text{Ans//}$$

$$\tan\theta = \frac{\Sigma F_V}{\Sigma F_H}$$

Q. A particle is acted upon by a three forces equal to 50N, 100N and 130N along the three sides of a equilateral triangle taken in order find magnitude and direction of resultant force.



Solution



$$\Sigma F_H = +50KN - 100\cos 60 - 130\cos 60$$
$$= 50KN - 100\times\frac{1}{2}KN - 130\times\frac{1}{2}$$
$$= \cancel{50KN} - \cancel{50} - 65$$
$$= -65KN.$$

$$\Sigma F_V = +100\cos 30 - 130\cos 30$$
$$= 100\times\frac{\sqrt{3}}{2} - 130\times\frac{\sqrt{3}}{2}$$
$$= -25.98KN$$

∴ Magnitude of Resultant force (F_R) :—

$$F_R = \sqrt{(\Sigma F_H)^2 + (\Sigma F_V)^2}$$

$$= \sqrt{(-65\,kN)^2 + (-25.98\,kN)^2}$$

$$= 69.9997\ kN$$

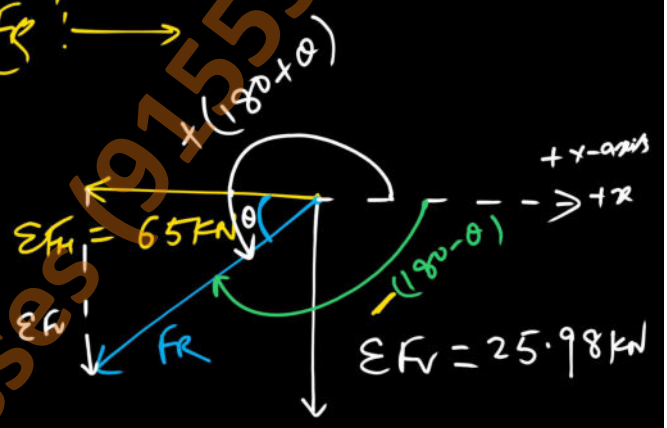
$$\approx 70\ kN \quad \text{Ans}$$

Direction of 'F_R' :→

$$\tan\theta = \frac{\Sigma F_V}{\Sigma F_H}$$

$$= \frac{25.98\,kN}{65\,kN}$$

$$\theta = 21.78^\circ$$



Direction of F_R $= +(180+\theta)$

$= +(180 + 21.78^\circ)$

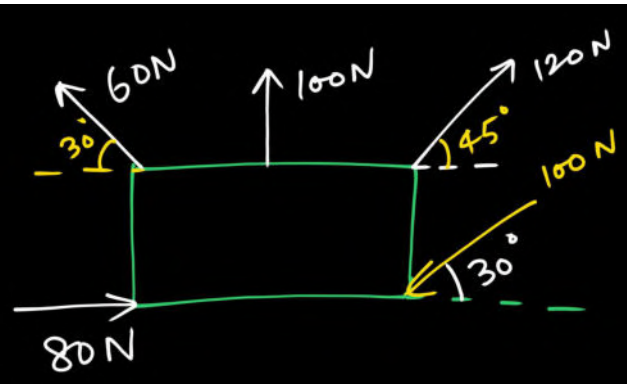
$= +201.78^\circ$ from x-axis

Again Direction of 'F_R' $= -(180-\theta)$

$= -(180 - 21.78)$

$= -158.22^\circ$ from x-axis

Q



Engg. Mechanics.

$$\Sigma F_H = -100\cos 30 + 120\cos 45 - 60\cos 30 + 80$$

$$= 26.29\ N.$$

$$\Sigma F_V = -100\cos 60 + 120\cos 45 + 100 + 60\cos 60$$

$$= 164.85\ N$$

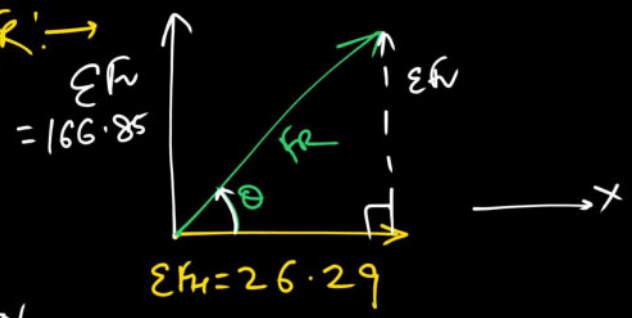
∴ Magnitude of Resultant force (F_R):→

$$F_R = \sqrt{(\Sigma F_H)^2 + (\Sigma F_V)^2}$$

$$= \sqrt{(26.29)^2 + (164.85N)^2}$$

$$\boxed{F_R = 166.93\ N}$$ Ans//

Direction of FR :→



$$\tan\theta = \frac{\Sigma F_V}{\Sigma F_H}$$

$$= \frac{166.85N}{26.29N}$$

$$\tan\theta = 6.346$$

$$\theta = \tan^{-1}(6.346) = 81.04^\circ \text{ Ans//}$$

Q// The followings forces acts at a point

(a) 20N inclined at 30° towards north of east

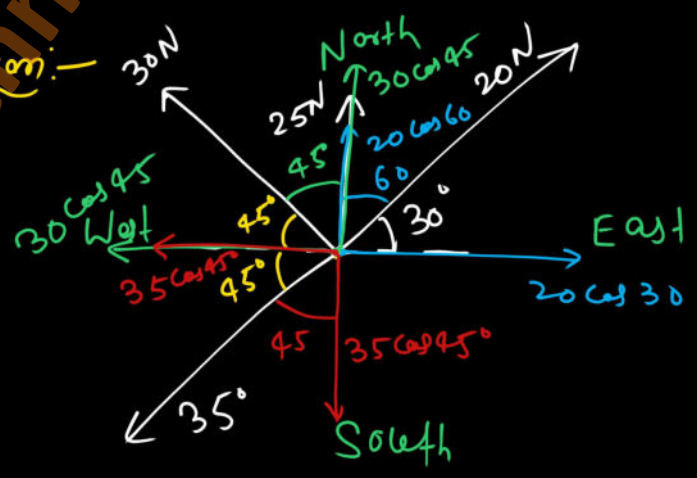
(b) 25N towards North

(c) 30N towards North of west

(d) 35N inclined 45° towards South of west

find Resultant force acting on the point.

Solution:-



∴

$$\Sigma F_H = 20\cos 30 - 35\cos 45 - 30\cos 45^\circ$$

$$= -28.64\,N$$

$$\Sigma F_V = +25 + 30\cos 45 + 20\cos 60 - 35\cos 45$$

$$= 31.46\,N$$

$\therefore$ Magnitude of resultant force (F_R):—

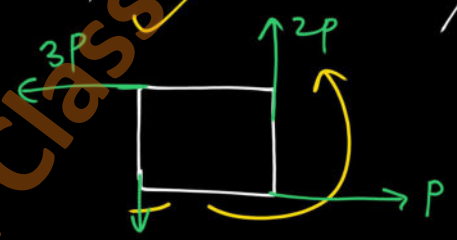
$$\because F_R = \sqrt{(\Sigma F_H)^2 + (\Sigma F_V)^2} = \sqrt{(-28.64\,N)^2 + (31.46)^2}$$

$$\boxed{F_R = 42.54\,N}$$ Ans//

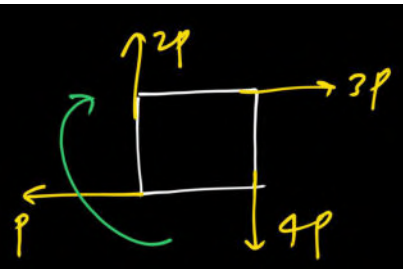
Q 1(a) Four forces P, 2P, 3P, and 4P acts along side, takes in order of a square, The force is ----

(zero, $2\sqrt{2}P$, 2P)

Solⁿ:—



or



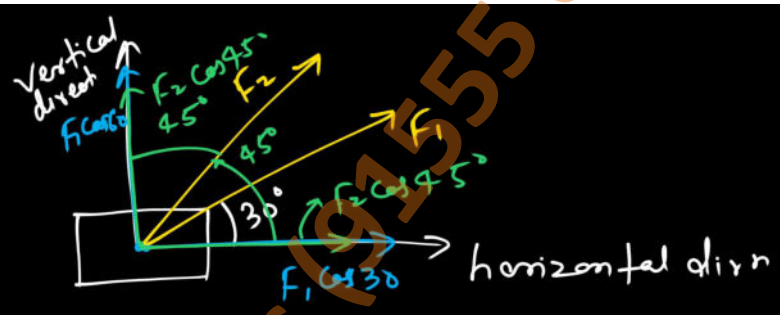
$$\Sigma F_H = +3P - P = 2P$$

$$\Sigma F_V = -4P + 2P = -2P$$

$$F_R = \sqrt{(\Sigma F_H)^2 + (\Sigma F_V)^2} = \sqrt{(2P)^2 + (2P)^2}$$

$$= \sqrt{4P^2 + 4P^2} = 2\sqrt{2}P$$ Ans

Q. Two forces $F_1 = 60N$ and $F_2 = 30N$ are acting on a body at an angle of $30°$ and $45°$ as shown in fig. Determine the total force acting in horizontal and vertical direction.



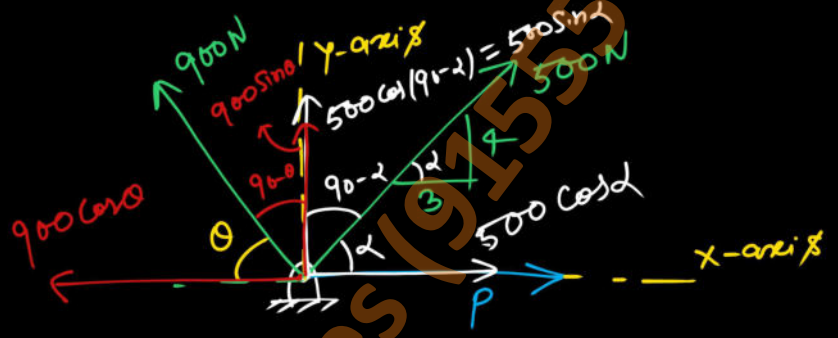
∴ Total force in horizontal direction:—

$$\Sigma F_H = F_1 \cos 30 + F_2 \cos 45$$
$$= 60 \cos 30 + 30 \cos 45$$
$$= 73.17 N \text{ Ans//}$$

Again total force acting on vertical direction:—

$$\Sigma F_V = F_1 \cos 60 + F_2 \cos 45°$$
$$= 60 \cos 60 + 30 \cos 45°$$
$$= 51.21 N \text{ Ans//}$$

Q. The resultant of three concurrent forces acting on a body as shown in fig. is 800 N in the direction of positive Y-axis. Determine the magnitude of force 'P' and angle θ.



Sol^n:— Given Resultant force (F_R) = 800 N

∴ Direction of F_R = positive Y-axis.

$$\tan\alpha = \frac{4}{3}$$

$$\alpha = \tan^{-1}(4/3) = 53.13^\circ \simeq 53^\circ$$

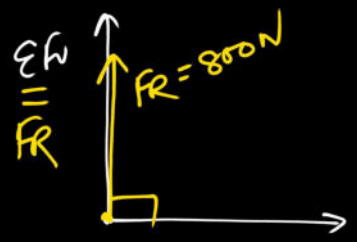
$$\because \Sigma F_H = 500\cos\alpha + P - 900\cos\theta$$

$$\cancel{0} = 500\cos 53 + P - 900\cos\theta \quad \text{—— (I)}$$

$$\Sigma F_V = +500\sin\alpha + 900\sin\theta$$

$$= 500\cos 53 + 900\sin\theta \quad \text{—— (II)}$$

$\therefore \Sigma F_V = F_R = 800N$



$500\cos 53 + 900\sin\theta = 800N$

$\Sigma F_H = 0$

$900\sin\theta = 800 - 500\cos 53$

$= 499.09$

$\sin\theta = \frac{499.09}{900}$

$\sin\theta = 0.554$

$\theta = \sin^{-1}(0.554)$

$\boxed{\theta = 33.67^\circ}$ Ans//

Again $\because \Sigma F_H = 0$

$500\cos 53 + P - 900\cos\theta = 0$

$500\cos 53 + P - 900\cos 33.67 = 0$

$P = 900\cos 33.67 - 500\cos 53 = 448.11N$ Ans//

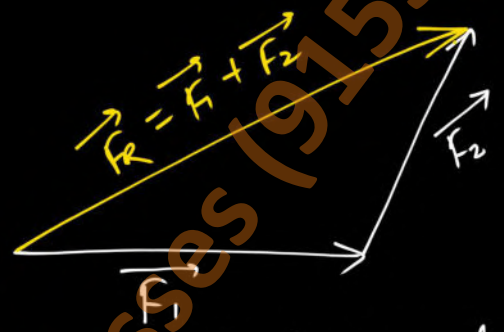
Geometrical Method for finding Resultant force :→

- Triangle law,
- polygon law.

Triangle law :→

→ This law states that if two forces are taken as two adjacent sides of a triangle in same order then third side of that triangle taken in opposite order will represent, resultant force of the two forces.

i.e

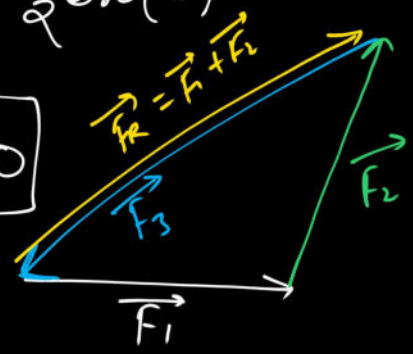


where, $\vec{F_R}$ = Resultant force of force $\vec{F_1}$ and $\vec{F_2}$.

Note** :→ Net resultant force of three forces forming a close triangle in same order is zero(0) and vice-versa.

Here

$$\boxed{\vec{F_1} + \vec{F_2} + \vec{F_3} = 0}$$



$\vec{F_3} = \vec{F_1} + \vec{F_2}$

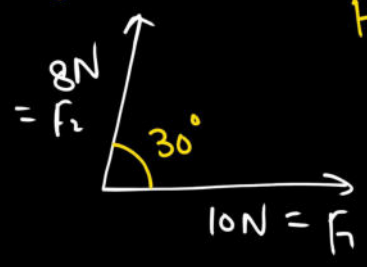
$-\vec{F_3} = \vec{F_R}$

$\therefore \vec{F_3} + \vec{F_R}$

$\Rightarrow \vec{F_R} + \vec{F_R} = 0$

Engg. Mechanics

Q Find the resultant force of given two forces By Geometrical method:



$$F_R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$F_R = \sqrt{}$$

$$= \underline{17.39} \text{ Ans//}$$

Sol^n:- By Geometrical Method:→

let a scale:—

1 cm = 1 N

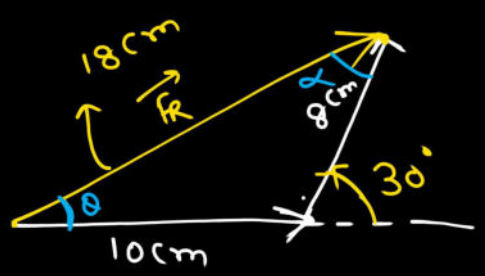
10 cm = 10 N

8 cm = 8 N

$$|\vec{F_R}| = 18 \times 1\,N$$

$$= 18\,N$$

Ans//

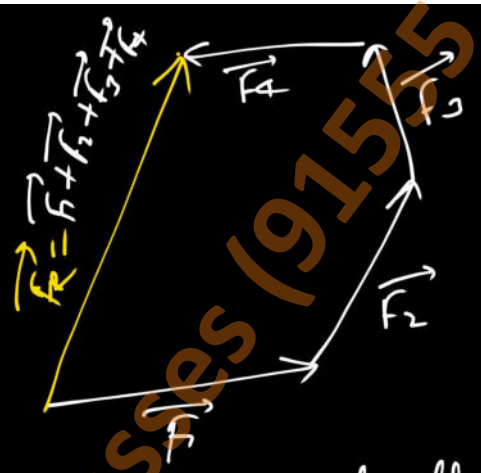


Direction of $\vec{F_R}$

θ → is the angle made by resultant force $\vec{F_R}$ with 10N force.

Polygon law :→

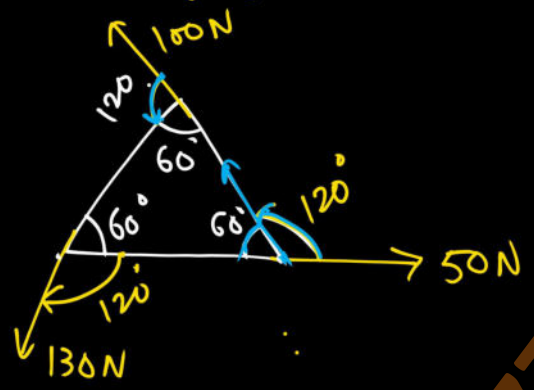
→ This law states if two or more than forces are added vectorally then take all forces as adjacent sides of a polygon in same order then last closing side of that polygon taken in opposite order will represent resultant force of all the forces.



where $\vec{F_R}$ = Resultant force of all the forces $\vec{F_1}$, $\vec{F_2}$, $\vec{F_3}$, and $\vec{F_4}$.

Q. find the resultant force of the given system forces by geometrical method:—

$\boxed{F_R = 70N}$



Solution:-

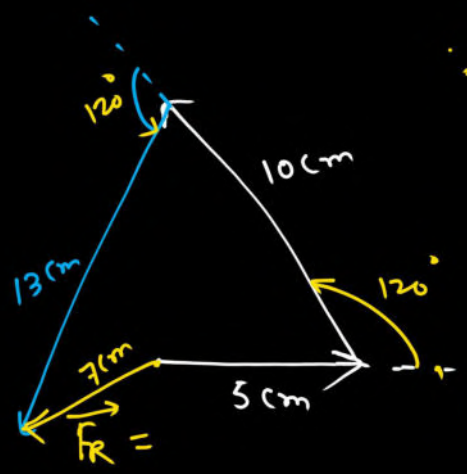
let a scale:—

10 N = 1 cm

∴ 50 N = 5 cm ✓

Again ∴ 100 N = 10 cm ✓

and Again ∴ 130 N = 13 cm ✓



∵ 1cm = 10N

7cm = 70N Ans//

FR.

Engg. Mechanics

UNIT-02- EQUILIBRIUM :

particle :→ A body or object or entity having negligible volume is calle particle

example, electron, proton, atom, ant etc.

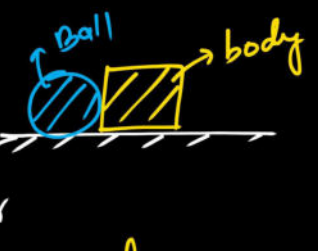
Rigid body :→

→ A body is said to be a rigid body if distance between two points on the body remains constant or if deformation in the does not take place after application of certain amount of external deforming force.

Free body :→

→ If a body is isolated from the objects/bodies which are in contact with the body seperately for analysis then body is called free body.

example:



*****

free body diagram (FBD) :→

→ If a body isolated from its contacting bodies and all the forces acting on the body are shown with the help of a vector arrow diagram, this diagram is called free body diagram.

→ followings types of forces can acts on a body: →

ⓐ weight of the body. (mg)

ⓑ Normal Reaction (N or R).

ⓒ External force on the body.

ⓐ weight of a body: →

→ This is the gravitational force applied by earth on the body:-

$$\text{weight}\ \boxed{(w) = mg}$$

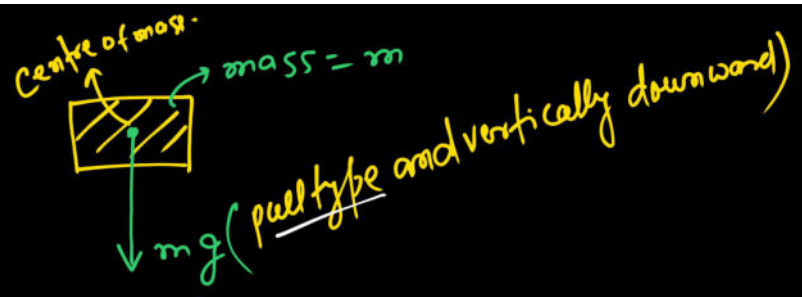
where m = mass of the body (kg)

g = acceleration due to gravity

$= 9.81\ \frac{m}{sec^2} \simeq 10\ \frac{m}{sec^2}$

**

→ weight of a body is always pull type and act at its centre of mass vertically downward.

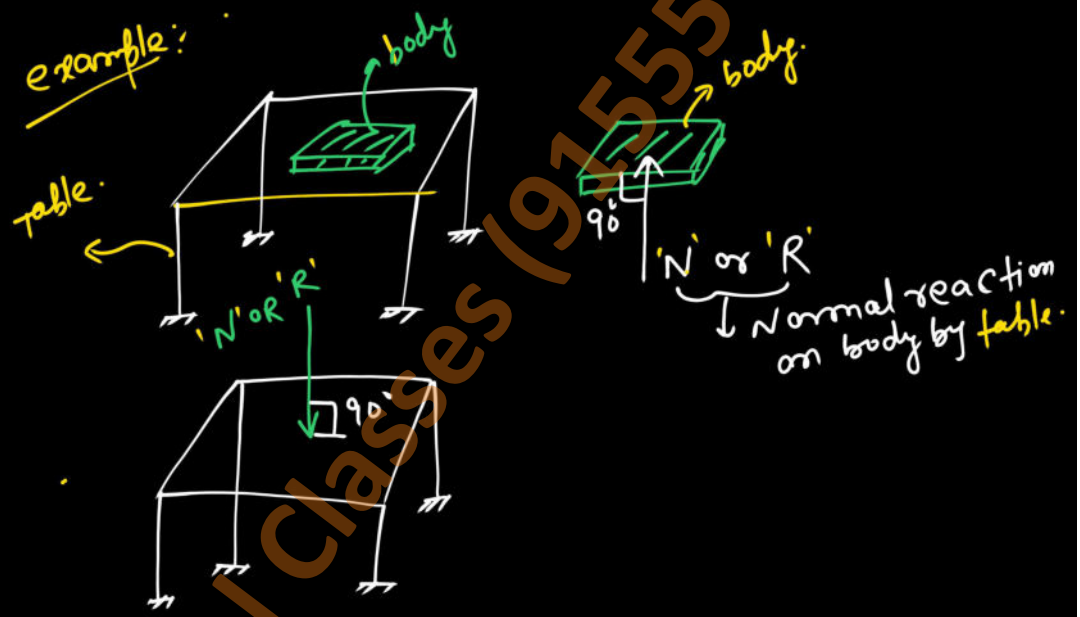


(5) Normal reaction :→

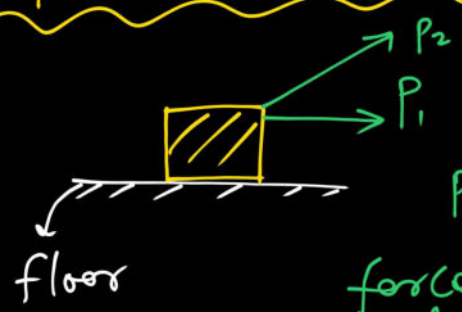
perpendicular.

→ when two bodies are in contact then one body apply a force perpendicular to surface of another body and push type in nature, this called normal reaction.

example:



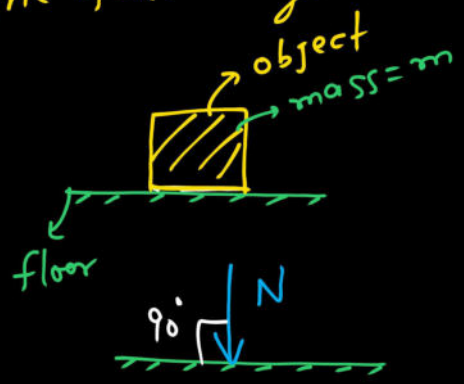
External force on the object :→



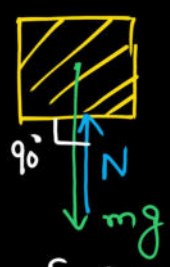
P_1, P_2 → external force applied by external objects/person.

Q Draw free body diagram (FBD) of the following:—

(1)



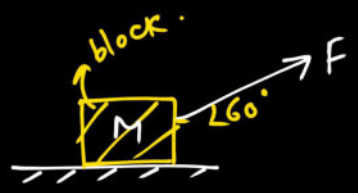
F.B.D of floor

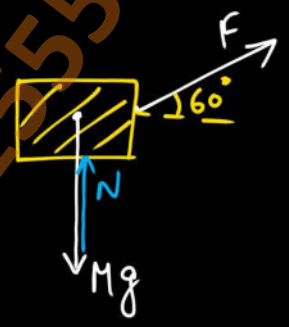


N = Normal reaction by floor on the object

F.B.D of object

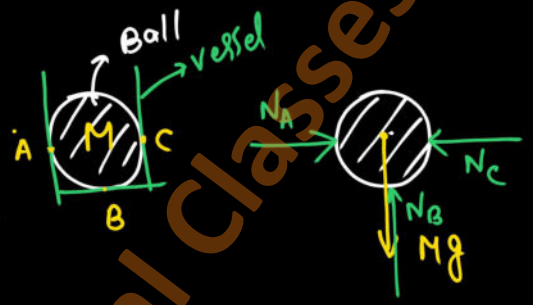
(2)





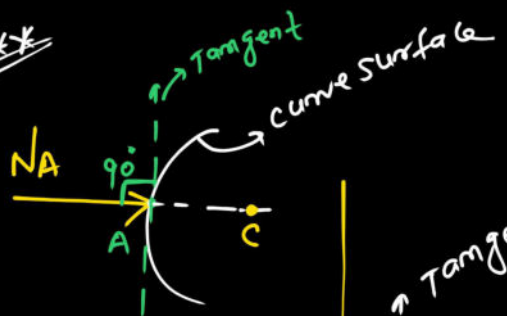
F.B.D of block

Q



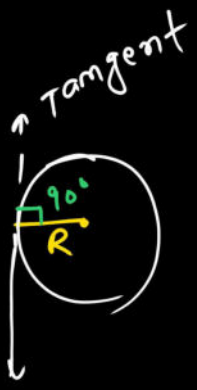
$N_A, N_B, N_C \rightarrow$ normal reaction at A, B and C by vessel.

Note:— (a) ***

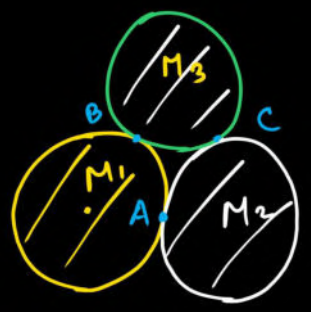


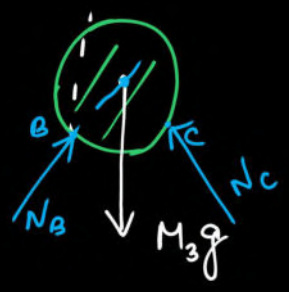
***

→ perpendicular to any curve surface always passes through centre of that curve surface



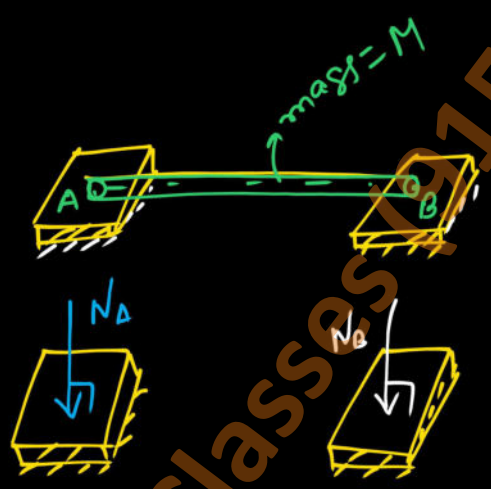
Q

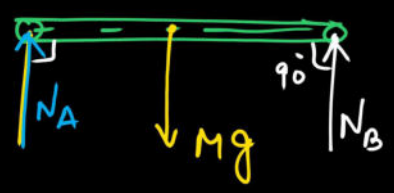




→ H.W =

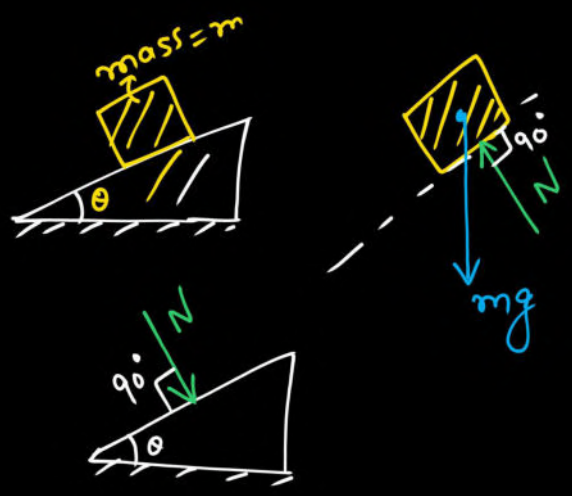
Q



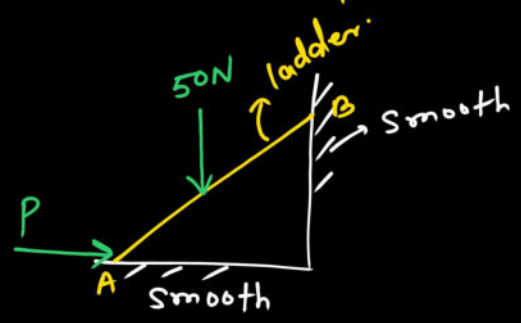


F. B. D

Q

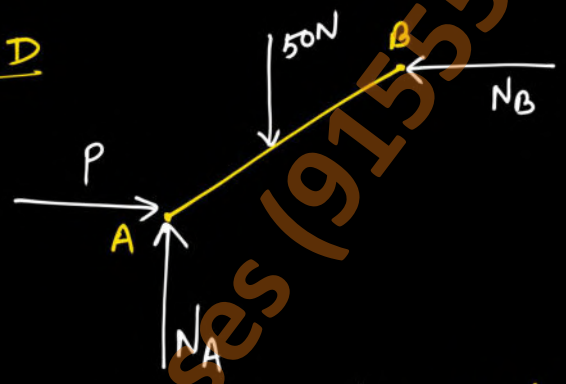


Q. Draw F.B.D of the ladder shown in fig.



Smooth → friction is zero.

F.B.D



N_A, N_B → Normal reaction at 'A' and B.

Engg. Mechanics

Equilibrium :→

→ Equilibrium is a state of an object in which object is in state of rest or uniform motion (constant velocity) along a straight line.

Required and sufficient condition of equilibrium :→

→ There are two condition simultaneously to be in equilibrium of an object :→

(a) Net/Total/Resultant force must be zero. $\boxed{\vec{F_R} = 0}$ where F_R → total/Resultant force.

(b) Net/Total/Resultant moment must be zero $\boxed{\vec{M_R} = 0}$ M_R → Resultant moment.

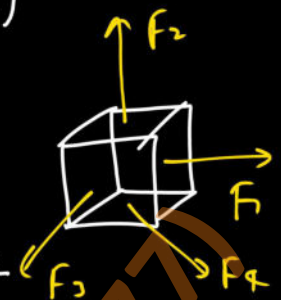
i.e for equilibrium (eqbm)

(a) $\boxed{F_R = 0 \text{ and } (b)\ M_R = 0}$

(a) Net/Resultant $(F_R) = 0$

$$\because F_R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

for 3-Dimension forces :-



$$\because F_R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2 + (\Sigma F_z)^2}$$

For eqbm :- $F_R = 0$

$$F_R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2 + (\Sigma F_z)^2} = 0$$

$$\Rightarrow (\Sigma F_x)^2 + (\Sigma F_y)^2 + (\Sigma F_z)^2 = 0$$

$\therefore$ $\Sigma F_x = 0$ — (I)

and $\Sigma F_y = 0$ — (II)

and $\Sigma F_z = 0$ — (III)

→ Simultaneously

(b) Net/Resultant moment $(M_R) = 0$

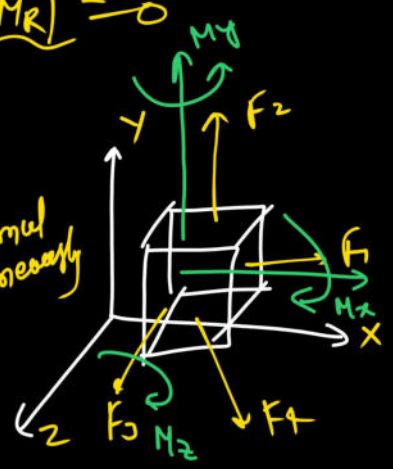
$$\because \vec{M}_R = 0$$

$\Sigma M_x = 0$ — (IV)

$\Sigma M_y = 0$ — (V)

$\Sigma M_z = 0$ — (VI)

→ Simultaneously



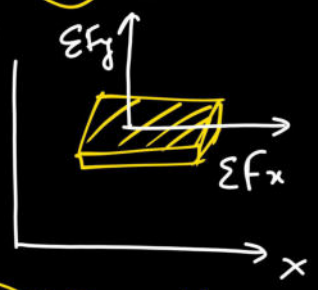
**** → Above six equations are called equations of equilibrium in 3-D.

→ Above six equations must satisfy simultaneously to be in equilibrium in 3-D of a object.

Equilibrium in a plane or 2-D :→

$$\therefore \boxed{\begin{aligned} \Sigma F_x &= 0 \quad \text{(I)} \\ \Sigma F_y &= 0 \quad \text{(II)} \\ \Sigma M_z &= 0 \quad \text{(III)} \end{aligned}}$$

→ Simultaneously



→ Above three equation must satisfy simultaneously to be in equilibrium in 2-D (plane) of an object.

→ Above three equations are called equations of equilibrium in a plane (2-D).

Note :→

(a) $\Sigma F_x = \Sigma F_H$ → All forces in horizontal direction

ΣF_x → All forces in x-direction

(b) $\Sigma F_y = \Sigma F_V$ → All forces in vertical direction.

ΣF_y → All forces in y-direction

Equilibrant :→

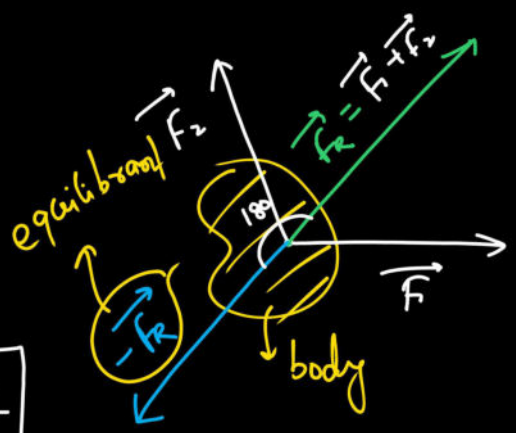
→ When number of concurrent forces acting on a body, then a single force which keeps the body in equilibrium is called equilibrant.

** → Magnitude of equilibrant is always equal to magnitude of resultant force and its direction is always opposite to direction of resultant force.

Net force on body $= F_R' = F_R - F_R$

$= F_R' = 0$

$\boxed{-\vec{F_R} = \text{Equilibrant}}$



Q. Two forces 8N and 12N act on a particle and their line of action are inclined at an angle of 120° find third force which can keep the particle in equilibrium.

Solution:

$\vec{F_R}$ = Resultant of two forces '8N' and '12N'

$-\vec{F_R}$ = Equilibrant = Third force which can keeps the particle equilibrium.

$$\therefore F_R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$= \sqrt{(8N)^2 + (12N)^2 + 2\times8\times12\cos120^\circ}$$

$$= 10.58\ N$$

∴ A third force of magnitude 10.58N can keep the particle in equilibrium Ans:

Direction of third force/equilibrant :—

$$\tan\alpha = \frac{8 \times \sin 120}{12 + 8\cos 120} = \frac{6.928}{12 + (-4)}$$

$$= 0.866$$

$$\alpha = \tan^{-1}(0.866)$$

$$\alpha = 40.89^\circ$$

$\therefore$ Direction of equilibrant $= (180 - \alpha) = 180 - 40.89$

$= 139.11^\circ$ from 12N clockwise force.

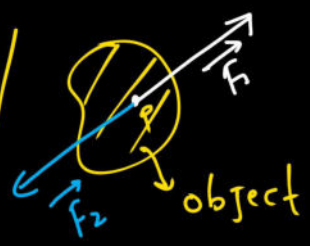
Engg. Mechanics

Equilibrium under Concurrent and Coplanar forces :→

** → For the case of Concurrent and Coplanar system of forces, if net/total/resultant force is zero, then total moment (M_R) automatically becomes zero, by Varignon's principle.

Case – (a) Equilibrium under two concurrent forces :—

$$|\vec{F_1}| = |\vec{F_2}|$$



$$F_R = 0$$

$$F_1 - F_2 = 0$$

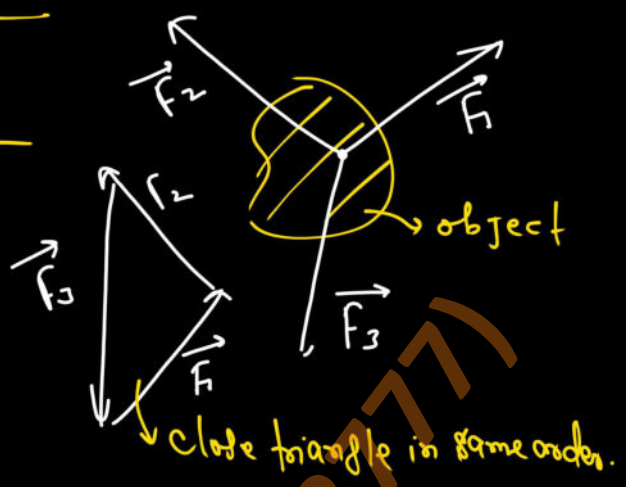
$F_1 = F_2 =$ magnitude of Both forces must be equal and direction should be opposite.

Case-b :- Equilibrium under three concurrent forces :—

∵ object eqbm :—

∴ $F_R = 0$

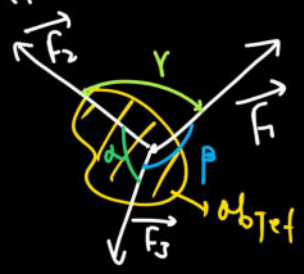
$$\vec{F_1} + \vec{F_2} + \vec{F_3} = 0$$



*** # Lame's Theorem (Sine Rule) :→

→ If a body/object is in equilibrium under action of three coplanar and concurrent forces then relation between them is given by

$$\therefore \boxed{\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}}$$



where α = angle between $\vec{F_2}$ and $\vec{F_3}$

β = " " $\vec{F_1}$ and $\vec{F_3}$

γ = " " $\vec{F_2}$ and $\vec{F_1}$.

$|\vec{F_1}| = F_1$ = Magnitude of $\vec{F_1}$

$|\vec{F_2}| = F_2$ = " " $\vec{F_2}$

$|\vec{F_3}| = F_3$ = " " $\vec{F_3}$

Proof :→

$\because F_R = 0$

$\vec{F_1} + \vec{F_2} + \vec{F_3} = 0$

$\vec{F_3} = -(\vec{F_1} + \vec{F_2})$

$\vec{F_3} = -\vec{F_R}$

$|\vec{F_3}| = |\vec{F_R}| = \text{same} = F_3 = F_R$

where $\vec{F_R} = \vec{F_1} + \vec{F_2}$ = Resultant force of $\vec{F_1}$ and $\vec{F_2}$.

$\because$ in triangle $\triangle OMN$:—

$(180-\alpha) + (180-\beta) + \theta = 180^\circ$

$\theta = \cancel{180} - \cancel{180} + \alpha - 180 + \beta$

$\boxed{\theta = \alpha + \beta - 180}$ — (1)

Again

$\because \alpha + \beta + \gamma = 360^\circ$

$\alpha + \beta = 360 - \gamma$

putting the above value of $(\alpha+\beta)$ in eqn (1) we get

$\because \theta = \alpha + \beta - 180$

$\boxed{\theta = 360 - \gamma - 180 = 180 - \gamma}$

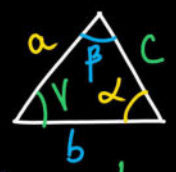
Apply Sine's law in $\triangle OMN$

$$\frac{F_1}{\sin(180-\alpha)} = \frac{F_2}{\sin(180-\beta)} = \frac{F_R}{\sin\theta}$$

$$\Rightarrow \frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin(180-\gamma)}$$

$$\Rightarrow \boxed{\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}}$$ proved//

Sine law in a triangle



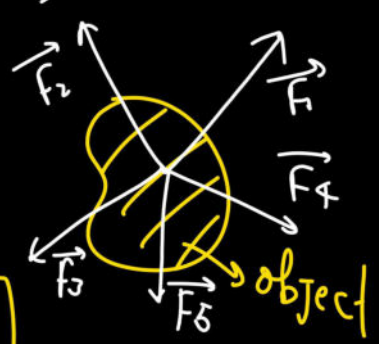
$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

Engg. Mechanics

Case-C :→ Equilibrium under number of concurrent forces :→

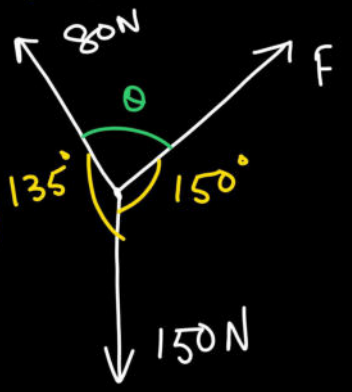
$\therefore \vec{F_R}$ = Resultant force of all concurrent forces = 0

i.e $\boxed{\Sigma F_x = 0}$ and $\boxed{\Sigma F_y = 0}$



Q. Find force 'F' in the followings system of forces.

Sol^n:- 1st Method



$\theta = 360° - (150 + 135)$

$= 360 - 285$

$= 75°$

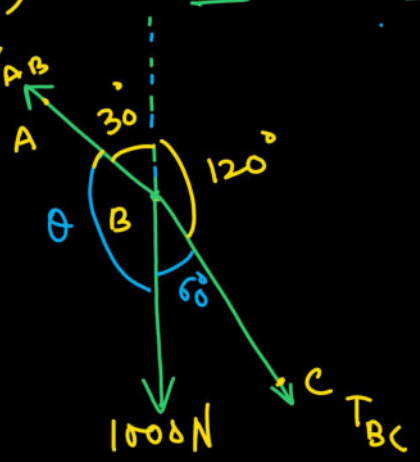
By Sine Rule:—

$$\frac{F}{\sin 135} = \frac{80N}{\sin 150} = \frac{150N}{\sin\theta} = \frac{150N}{\sin 75}$$

$$\because \frac{F}{\sin 135} = \frac{80N}{\sin 150}$$

$$\Rightarrow F = \sin 135 \times \frac{80N}{\sin 150} = 113.13N \text{ Ans}$$

Q. Find the tension (T) in 'AB' and 'BC' as shown in fig.



$\theta = 360° - (60 + 120 + 30)$

$= 360° - 210°$

$= 150°$

By lame's theorem:→

$$\frac{T_{AB}}{\sin 60} = \frac{T_{BC}}{\sin\theta} = \frac{1000\ N}{\sin 150}$$

$$\because \quad \frac{T_{AB}}{\sin 60} = \frac{1000}{\sin 150}$$

$$\Rightarrow \quad T_{AB} = \frac{\sin 60 \times 1000\ N}{\sin 150} = 1732.05\ N \text{ Ans}$$

Again $\frac{T_{BC}}{\sin\theta} = \frac{1000\ N}{\sin 150}$

$$\Rightarrow \quad \frac{T_{BC}}{\cancel{\sin 150}} = \frac{1000\ N}{\cancel{\sin 150}}$$

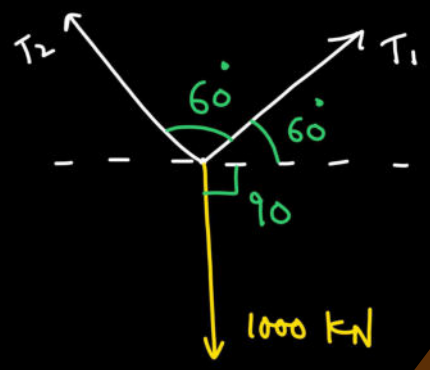
$$\Rightarrow \quad \boxed{T_{BC} = 1000\ N} \text{ Ans}$$

Engg. Mechanics

Q Find tension T_1 and T_2 for following fig.

$T_1 = ?$

$T_2 = ?$

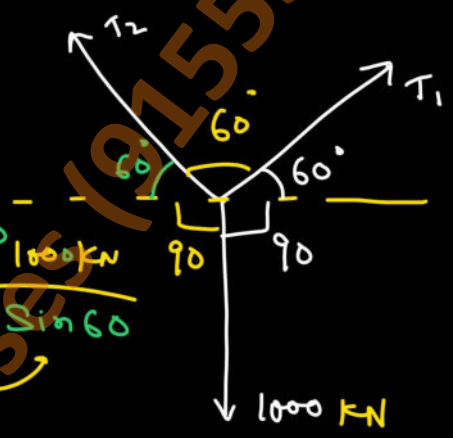


Solⁿ:

lame's Theorem

$$\frac{T_1}{\sin 150} = \frac{T_2}{\sin 150} = \frac{1000\,KN}{\sin 60}$$

$$\therefore \frac{T_1}{\sin 150} = \frac{1000\,KN}{\sin 60}$$



$$T_1 = \frac{1000\,KN \times \sin 150}{\sin 60} = 577.35\,KN$$

Ans//

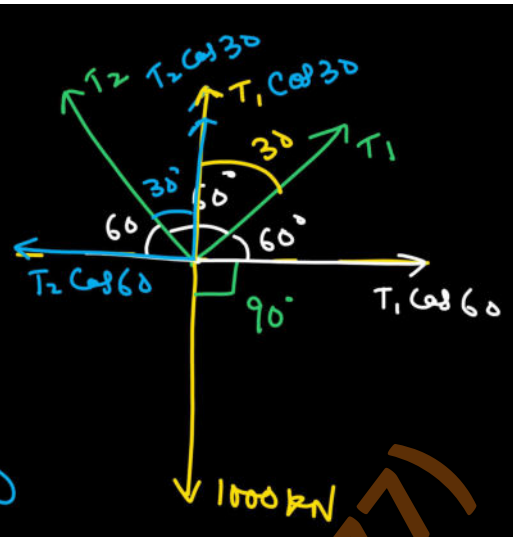
Again $\frac{T_2}{\sin 150} = \frac{1000\,KN}{\sin 60}$

$$\Rightarrow T_2 = \frac{1000\,KN \times \sin 150}{\sin 60} = 577.35\,KN$$

Ans//

2nd Method :-

$\Sigma F_x = 0$ and $\Sigma F_y = 0$



$\because \Sigma F_x = 0$

$+T_1 \cos 60 - T_2 \cos 60 = 0$

$\Rightarrow \boxed{T_1 = T_2}$ — (1)

$\because \Sigma F_y = 0$

$+T_1 \cos 30 + T_2 \cos 30 - 1000\,KN = 0$

$T_2 \cos 30 + T_2 \cos 30 = 1000\,KN$ [from eqn (1) $T_1 = T_2$]

$2 T_2 \cos 30 = 1000\,KN$

$T_2 = \dfrac{1000\,KN}{2 \times \cos 30} = 577.35\,KN$ Ans//

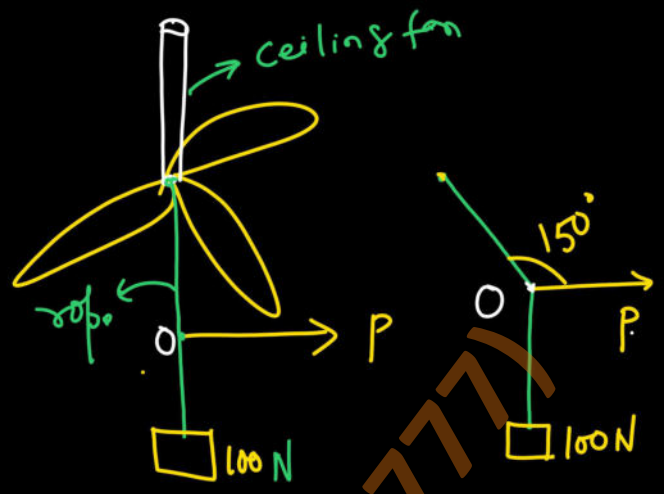
$\therefore T_1 = T_2 = 577.35\,KN$ Ans//

Q// A weight of 100 N is hung by a means of a rope to a hook in Ceiling fan Find (i) pull 'P' which is applied in horizontal direction which will make the rope 150° to the direction of pull.

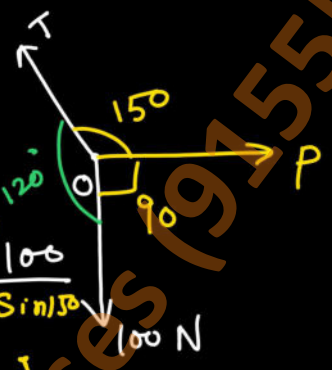
(ii) Minimum pull required to hold the rope in above position.

Calculate the tension in the rope in both the case.

Solution



By lame's Theorem:



$$\frac{T}{\sin 90} = \frac{P}{\sin 120} = \frac{100}{\sin 150}$$

$$\because \frac{T}{\sin 90} = \frac{100}{\sin 150}$$

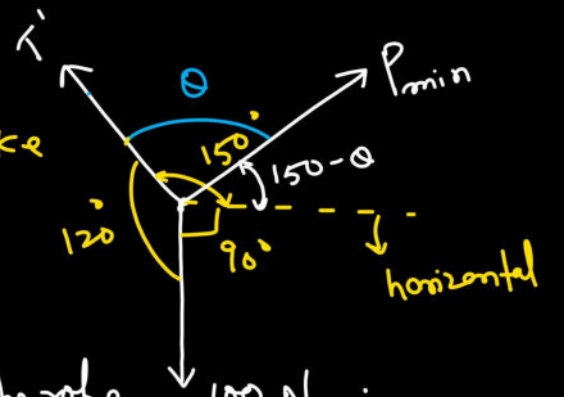
$$\Rightarrow T = \frac{100\,N \times \sin 90}{\sin 150} = 200\,N$$

Again $\because \frac{P}{\sin 120} = \frac{100\,N}{\sin 150}$

$$\Rightarrow P = \frac{100\,N \times \sin 120}{\sin 150} = 173.20\,N$$

Ans//

(II) Above position means rope will make same angle 150° with horizontal.



let T' = Tension in the rope.

θ = angle between minimum pull and T'.

∴ By lame's Theorem :-

$$\frac{T'}{\sin(240-\theta)} = \frac{P_{min}}{\sin 120} = \frac{100}{\sin\theta}$$

$$\therefore \frac{P_{min}}{\sin 120} = \frac{100}{\sin\theta}$$

$$\Rightarrow P_{min} = \frac{100 \times \sin 120}{\sin\theta}$$ Max^m

for P_{min}, $\sin\theta$ = Max^m = 1

$\therefore \boxed{\theta = 90°}$

$$\therefore P_{min} = \frac{100 \times \sin 120}{\sin 90}$$

$$= 100 \times \sin 120°$$

$$= 86.60 \text{ N}$$ Ans//

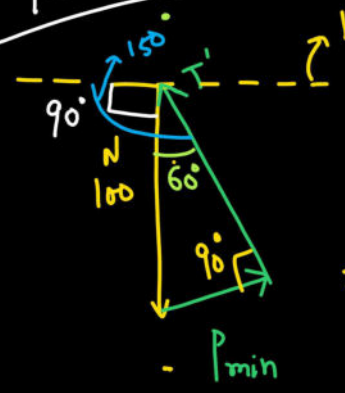
$$\because \frac{T'}{\sin(120-\theta)} = \frac{100}{\sin\theta}$$

$$\Rightarrow T' = \frac{100\sin(120-90)}{\sin 90}$$

$$= \frac{100\times\sin 30}{1} = 50\,N \text{ Ans//}$$

2nd Method

Geometrical Method

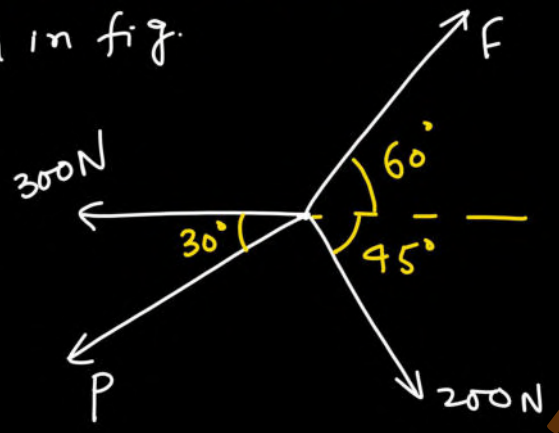


$$\sin 60 = \frac{P_{min}}{100\,N}$$

$$\Rightarrow P_{min} = 100\times\sin 60$$

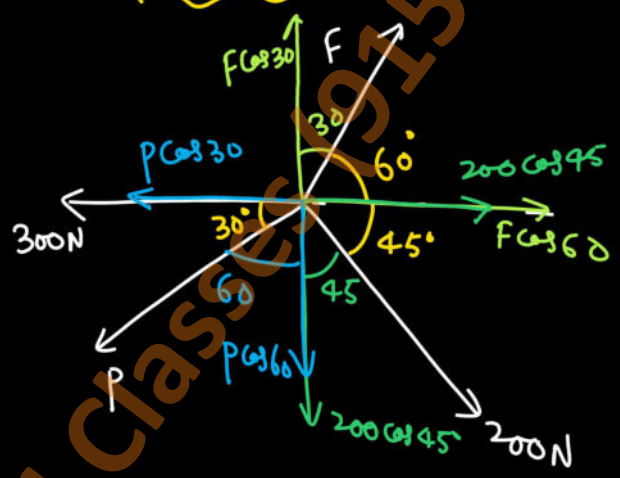
$$= 86.60\,N \text{ Ans//}$$

Q. Find the value of force 'F' and 'P' shown in fig.



Engg. Mechanics

Q.



$\because \Sigma F_x = 0$

$F\cos 60 + 200\cos 45 - 300 - P\cos 30 = 0$

$F \times \frac{1}{2} + 200 \times \frac{1}{\sqrt{2}} - 300 - P \times \frac{\sqrt{3}}{2} = 0$

$\Rightarrow \frac{F}{2} - 158.58 - \frac{P\sqrt{3}}{2} = 0$

$\Rightarrow \frac{F}{2} - \frac{\sqrt{3}P}{2} = 158.58$

$\Rightarrow F - \sqrt{3}P = 317.16$ —— (1)

$\because \Sigma F_y = 0$

$F\cos 30 - P\cos 60 - 200\cos 45 = 0$

$F \times \frac{\sqrt{3}}{2} - P \times \frac{1}{2} = 200 \times \frac{1}{\sqrt{2}}$

$\frac{\sqrt{3}F}{2} - \frac{P}{2} = 141.42$

$\Rightarrow \sqrt{3}F - P = 282.84$ —— (II)

$\because F - \sqrt{3}P = 317.16$ —— (I) $\times\sqrt{3}$

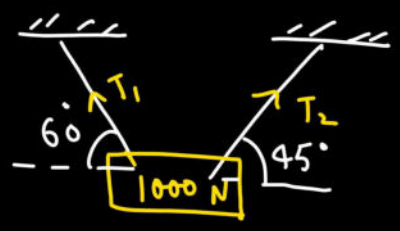
$-\sqrt{3}F \mp P = 282.84$ —— (II)

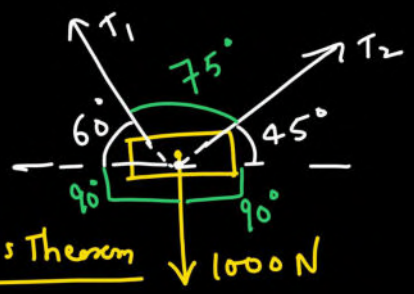
$-2P = 266.5$

$P = \frac{-266.5}{2} = -133.24$

Ans//

Q/ Find the tension T_1 and T_2 in the rope shown in fig.





$\Sigma F_x = 0$
$\Sigma F_y = 0$

By lame's Theorem

$$\frac{T_1}{\sin(135)} = \frac{T_2}{\sin(150)} = \frac{1000\,N}{\sin 75^\circ}$$

$$\Rightarrow \because \frac{T_1}{\sin 135} = \frac{1000\,N}{\sin 75}$$

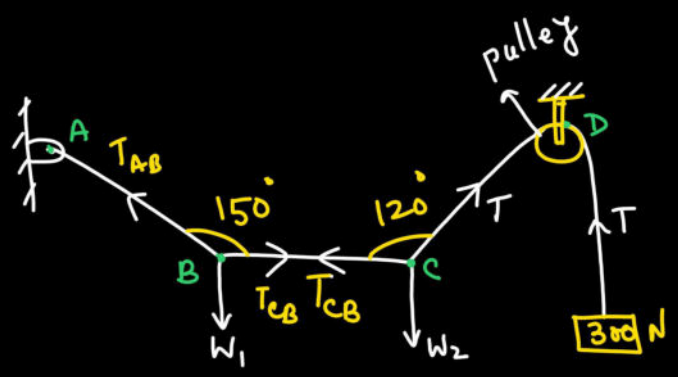
$$\Rightarrow T_1 = \frac{1000\,N \times \sin 135}{\sin 75} = 732.05\,N$$

Ans//

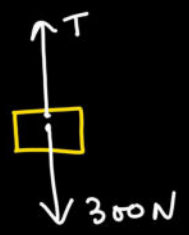
Again $\because \dfrac{T_2}{\sin 150} = \dfrac{1000\,N}{\sin 75}$

$$T_2 = \frac{1000\,N \times \sin 150}{\sin 75} = 517.64\,N$$ Ans//

Find weight 'W_1' and 'W_2' in shown in fig.



Solution:



$\Sigma F_x = 0$ $\boxed{0 = 0}$

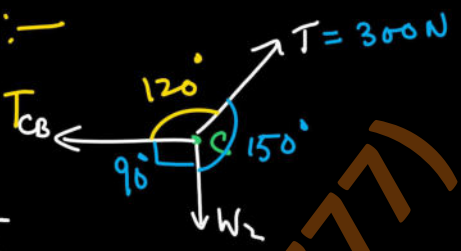
$\Sigma F_y = 0$

$+T - 300 = 0$

$\boxed{T = 300\ N}$

→ F.B.D of point 'C':—

By lame's Theorem:→



$$\frac{T_{CB}}{\sin 150} = \frac{W_2}{\sin 120} = \frac{300}{\sin 90}$$

$$\because \frac{T_{CB}}{\sin 150} = \frac{300\ N}{\sin 90}$$

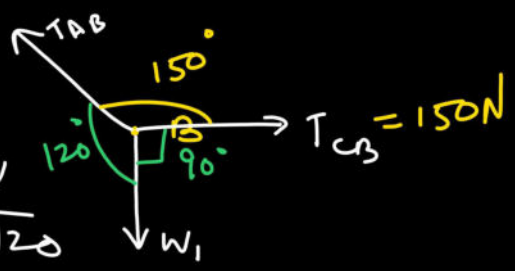
$$\Rightarrow T_{CB} = \frac{300\ N \times \sin 150}{\sin 90} = 150\ N$$

Again: $\frac{W_2}{\sin 120} = \frac{300\ N}{\sin 90}$

$$\Rightarrow W_2 = \frac{300 \times \sin 120}{\sin 90} = 259.80\ N \text{ Ans}$$

F.B.D of point 'B':

By lame's Theorem:—



$$\frac{T_{AB}}{\sin 90} = \frac{W_1}{\sin 150} = \frac{150\ N}{\sin 120}$$

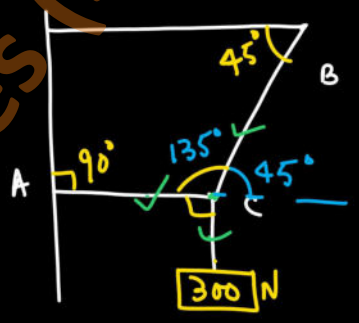
$$\therefore \frac{W_1}{\sin 150} = \frac{150\ N}{\sin 120}$$

$$\Rightarrow W_1 = \frac{150N \times \sin 150}{\sin 120} = 86.60N$$

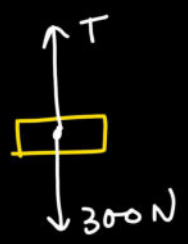
$$\therefore W_1 = 86.60N, \; W_2 = 259.80N \} \text{ Ans}$$

Q: A block of mass 30 kg is suspended by three strings shown in fig. find tension in each string.

Weight = mass × g
= 30 × 10
= 300 N



Sol^n

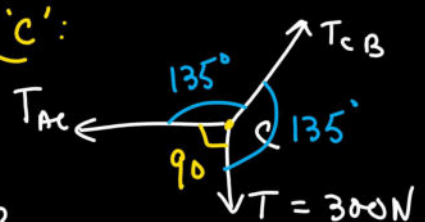


$\Sigma F_y = 0$

$T - 300N = 0$

$\boxed{T = 300\,N}$ Ans

Again F.B.D of point 'C':



$T_{AC} \rightarrow$ Tension in string AC

$T_{CB} \rightarrow$ Tension in string CB.

∴ By lame's Theorem :—

$$\frac{T_{AC}}{\sin 135} = \frac{T_{CB}}{\sin 90} = \frac{300\,N}{\sin 135}$$

$$\because \frac{T_{AC}}{\cancel{\sin 135}} = \frac{300\,N}{\cancel{\sin 135}}$$

$$\Rightarrow \boxed{T_{AC} = 300\,N}$$ Ans//

Again ∵ $\frac{T_{CB}}{\sin 90} = \frac{300\,N}{\sin 135}$

$$T_{CB} = \frac{300\,N \times \sin 90}{\sin 135} = 424.26\,N$$

Ans//

Q// A smooth sphere of weight 'W' is suppoted by a string fastened to a point 'A' on the smooth vertical wall the other end is in contact with point 'B' on the wall as shown in fig, If the length of string 'AC' is equal to radius of sphere find tension and reaction of wall in terms of W.

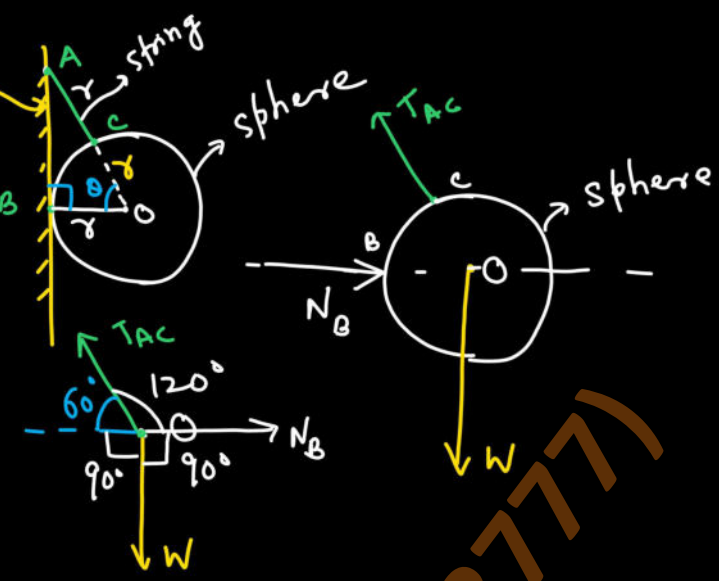
r = radius of sphere

$\because \triangle ABO$

$\cos\theta = \frac{\cancel{r}}{2\cancel{r}}$

$\cos\theta = \frac{1}{2}$

$\boxed{\theta = 60^\circ}$



By Lame's Theorem:—

$$\frac{N_B}{\sin(90+60)} = \frac{T_{AC}}{\sin 90} = \frac{W}{\sin 120^\circ}$$

$$\therefore \frac{N_B}{\sin 150} = \frac{W}{\sin 120}$$

$$N_B = \frac{W \times \sin 150}{\sin 120} = \underline{W \times 0.5773} \text{ Ans//}$$

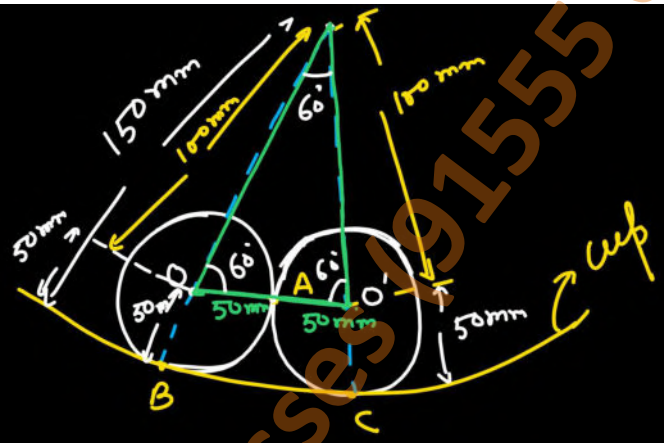
Again $\frac{T_{AC}}{\sin 90} = \frac{W}{\sin 120}$

$$\Rightarrow T_{AC} = \frac{W \times \sin 90}{\sin 120} = \underline{1.15\,W}$$

Ans//

T_{AC} ↓ Tension in string.

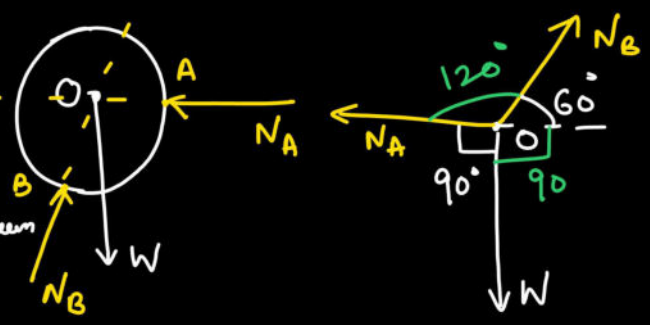
Q. Two equal heavy sphere of 50 mm radius are in equilibrium within a smooth cup of 150 mm radius shown in fig. Show that the reaction between the cup and one sphere is double than that between two sphere.



Solution :→ let W = weight of each sphere

N_B → Reaction by cup on sphere

N_A → Reaction between two sphere.



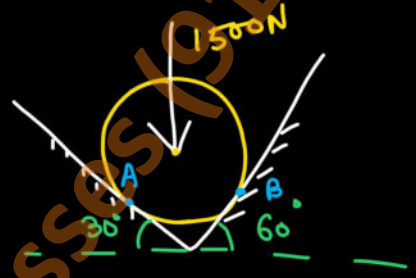
Lamme's Theorem at 'O'

$$\frac{N_A}{\sin 150^\circ} = \frac{N_B}{\sin 90^\circ} = \frac{W}{\sin 120^\circ}$$

$$\Rightarrow \because \quad N_A = \frac{N_B \times \sin 150}{\sin 90^\circ} = N_B \times \frac{1}{2}$$

$$\Rightarrow \quad N_B = 2 N_A$$ proved//

Q. Find the reactions at 'A' and 'B' shown in fig.



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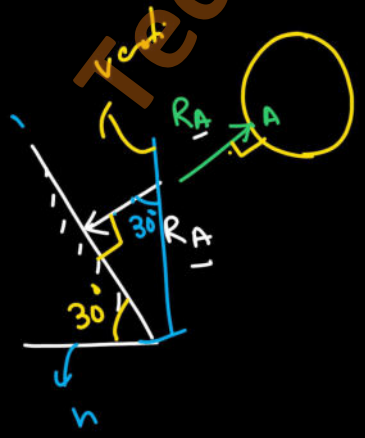
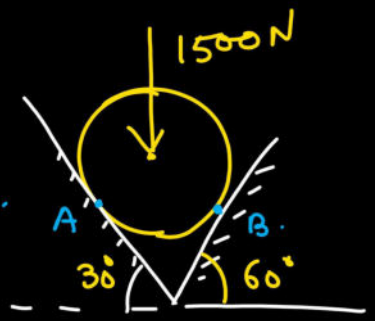
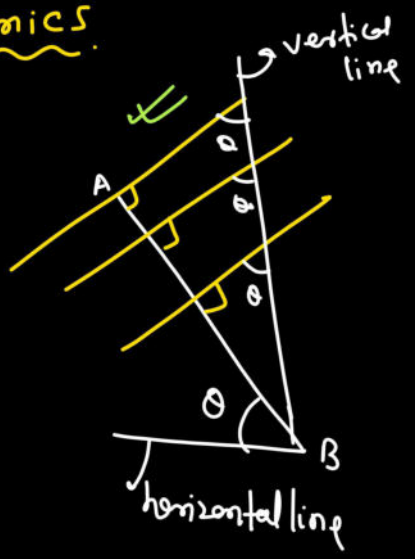
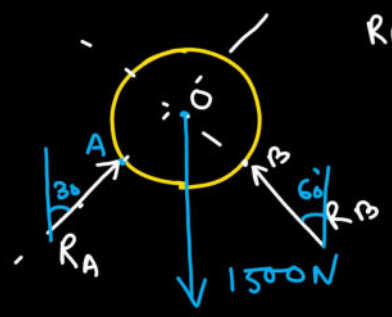
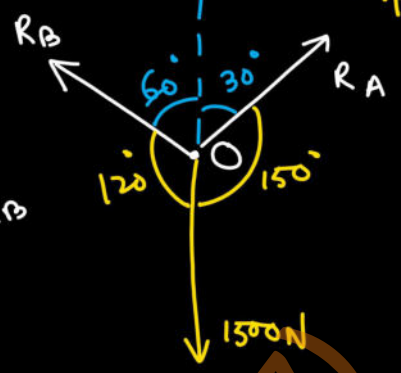






Sol^n :- let R_A and R_B are reaction at A and B resp.





∴ By lame's Theorem :-

$$\frac{R_A}{\sin 120} = \frac{R_B}{\sin 150} = \frac{1500\ N}{\sin 90}$$

$$\because \quad \frac{R_A}{\sin 120} = \frac{1500N}{\sin 90}$$

$$\Rightarrow \quad R_A = \frac{1500N \times \sin 120}{\sin 90} = \underline{1299.03\ N} \text{ Ans//}$$

Again $\frac{R_B}{\sin 150} = \frac{1500N}{\sin 90}$

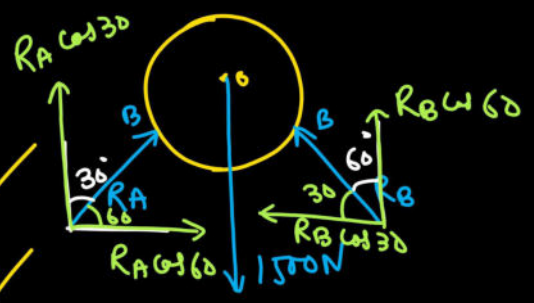
$$\Rightarrow \quad R_B = \frac{1500\ N}{\sin 90} \times \sin 150$$

$$= 750N \text{ Ans//}$$

2nd Method

$\Sigma F_x = 0$

$\Sigma F_y = 0$



$\Sigma F_x = 0$

$R_A \cos 60 - R_B \cos 30 = 0$ —— (I)

Again $\Sigma F_y = 0$

$R_A \cos 30 + R_B \cos 60 - 1500 = 0$

$R_A \cos 30 + R_B \cos 60 = 1500$ —— (II)

Solving Above equations, $R_A = 1299.03$, $R_B = 750N$ Ans//

Joints / supports :→

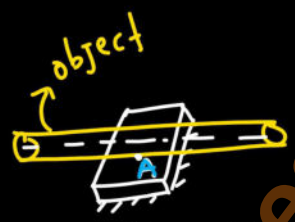
→ When two or more than two components/objects connected together, they form a Joint or provides support to each other.

Types of Joints/supports :→

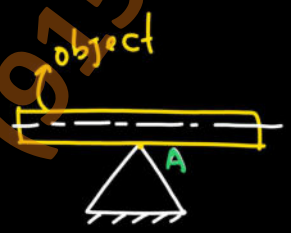
Followings are the important types of Joint:—

I Simply support
II Roller support
III Hinge Joint or Pin Joint.
IV fixed Joint.

I Simply-support :→

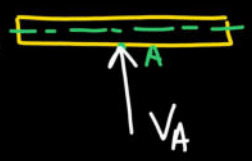


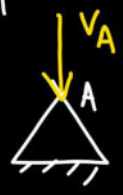
Simply-support



Simply support

V_A → vertical support reaction at A

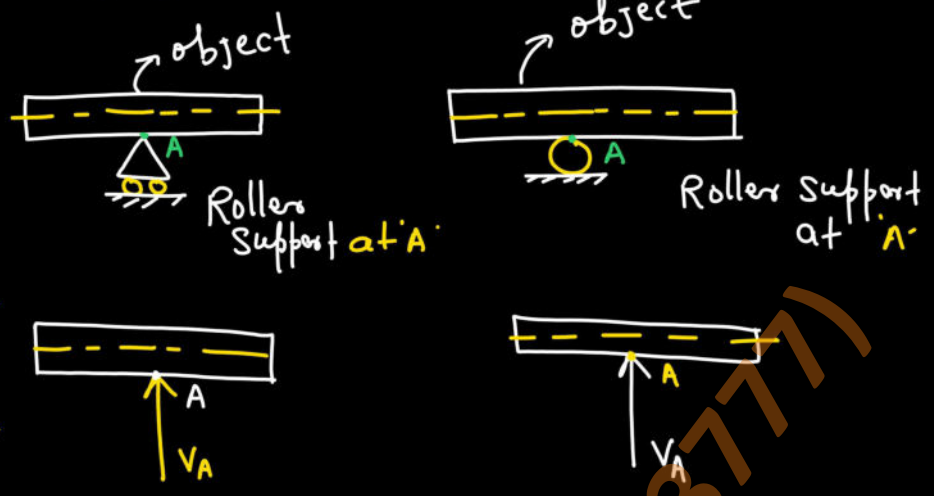


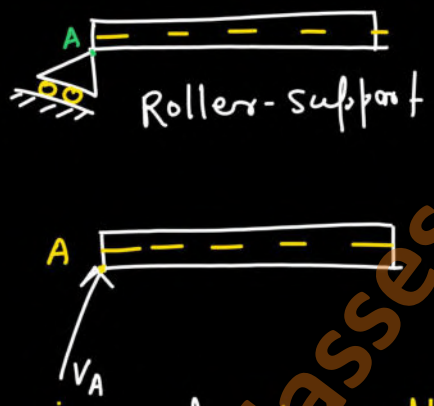


** → In case of simply-support Joint, there is only one support reaction (vertical force (V_A))

** → There is no supporting moment (resisting moment) at simply supported Joint

II Roller support :→



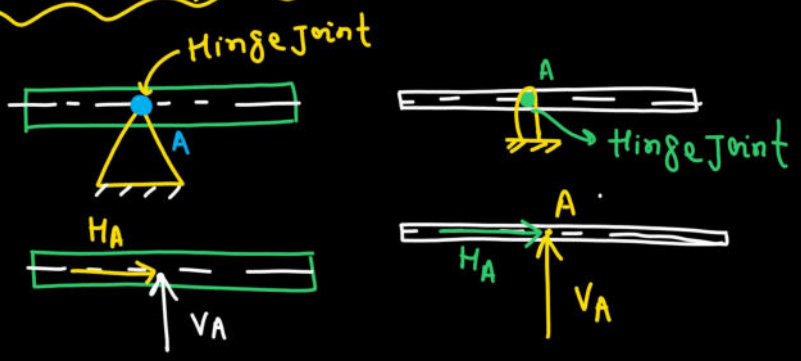


→ In case of roller-support, there only one support reaction (Vertical force) like-Simply-Support.

→ There is no supporting moment (resisting moment) at roller-support.

III Hinge Joint/Pin Joint :→

H_A → Horizontal Support reaction at 'A'



→ In case of Hinge or pin joint, there are two support reactions (Horizontal (H) and vertical force).

→ There is no supporting moment or resisting moment at hinge/pin joint

IV Fixed Joint :→

∴ M_A → Supporting or resisting moment at fixed joint A.

→ In case of fixed joint, there are three support reaction (Two forces and a resisting moment).

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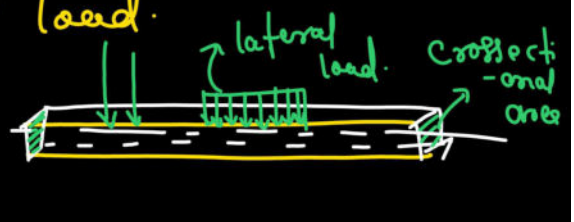
Beam :→ Beam is a horizontal structural member of uniform cross sectional area and supports transverse / lateral load.



Types of Beam :→

There are followings important types of Beam :—

I Simply-supported Beam.
II Continuous Beam.
III overhang Beam
IV Cantilever Beam
V propped cantilever Beam.

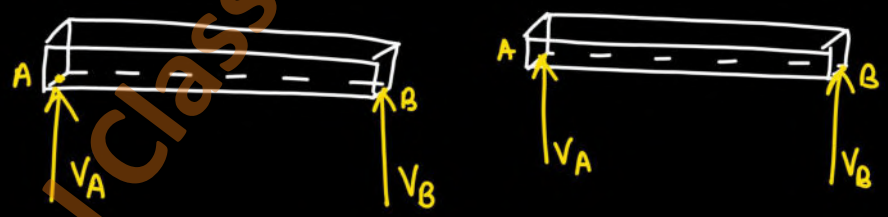
VI Fixed Beam.

I Simply-supported Beam :→

→ Those beam which both the ends are supported by either simply-support or roller support is called simply-supported Beam.

Simply supported Beam.

F.B.D:-



II Continuous Beam :→

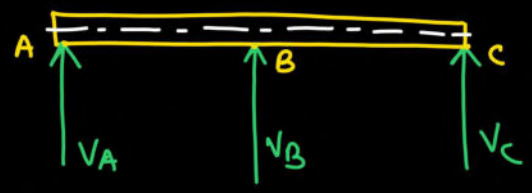
→ Those Beam which are simply or roller supported at more than two points on the beam including its two ends is called continuous beam.

example:

Continuous beam.

example: Continuous Beam.

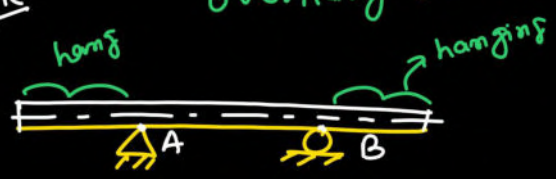
F.B.D:—



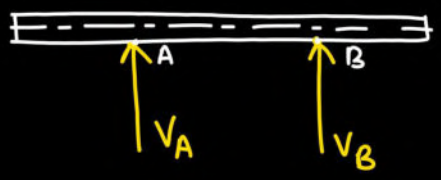
III overhang Beam :→

→ Those beams which some parts or length of either ends or both ends is hanging are called overhang Beam.

example overhang beam.



F.B.D:



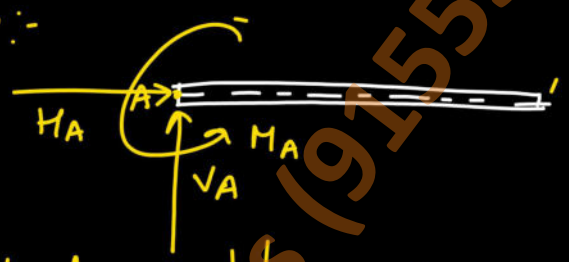
IV Cantilever Beam :→

Those beam which one end is fixed and end end is free (without any-support) is called cantilever beam or cantilever.

example:

Cantilever.

F.B.D :-



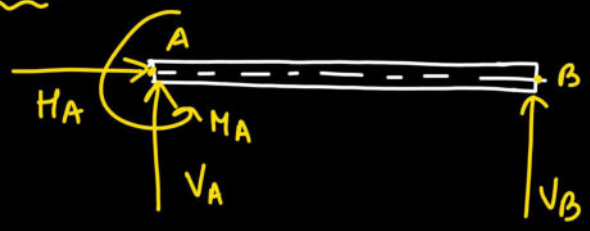
V propped Cantilever :→

→ A cantilever whose free end is either simply-supported or roller supported is called propped cantilever.

example

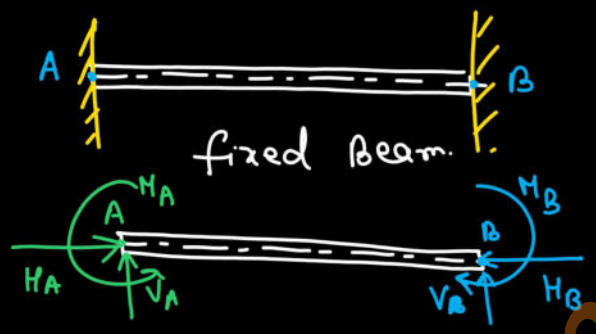
propped cantilever.

⇒ F.B.D:



VI Fixed Beam :→

Those Beams whose both ends are fixed, are called fixed Beam.



Load :→

→ Externally applied force and moment on the beam are called Load on the beam.

Types of Load :→

I Concentrated or point load.

II Uniformally distributed load (UDL)

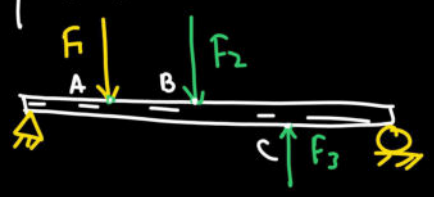
III Uniformally varying load (UVL)

IV Couple or moment.

I Concentrated point load :→

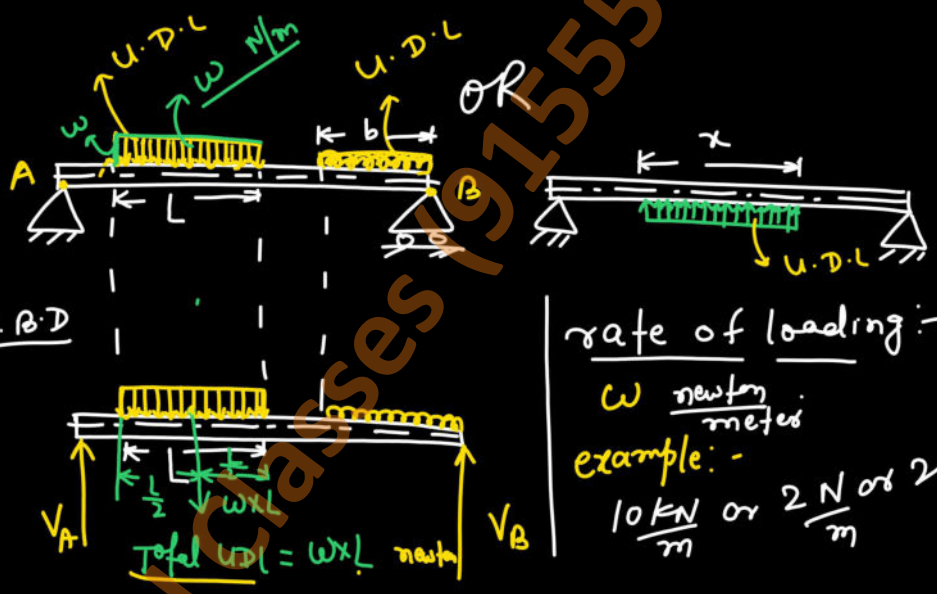
→ Externally applied force which act at a point on the beam is called concentrated or point load.

Ex :- F_1, F_2 and F_3 are point load.



II Uniformally Distributed Load (UDL) :→

→ Externally applied load which act on a span (length) of beam with constant rate of loading is called uniformally distributed load (UDL)



rate of loading :—

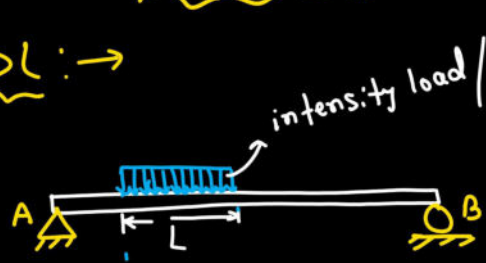
w $\frac{\text{newton}}{\text{meter}}$

example :-

$10\frac{kN}{m}$ or $2\frac{N}{m}$ or $20\frac{N}{m}$

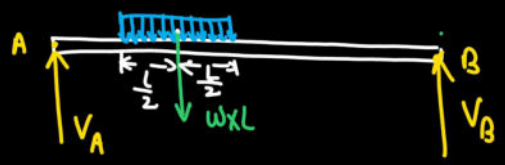
Engg. Mechanics.

II UDL :→



Total UDL = w×L newton.

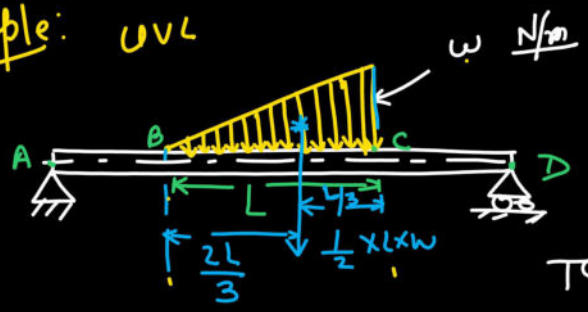
F.B.D



III uniformally varying load (UVL) :→

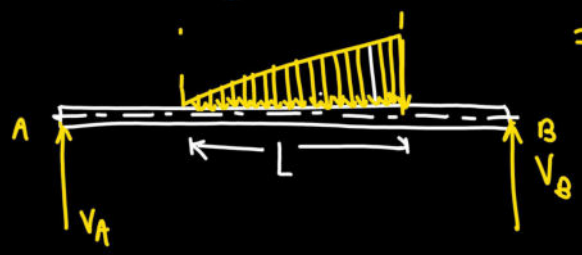
→ External load which acts over a span of beam and varying uniformally is called UVL.

example: UVL



Total UVL = $= \frac{1}{2} \times L \times w$ newton

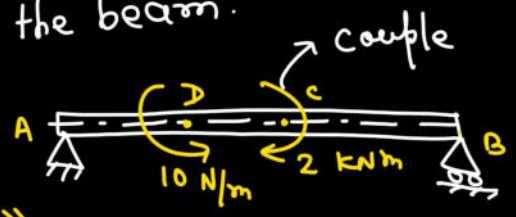
F.B.D



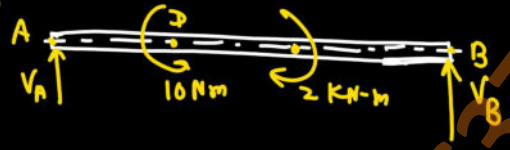
IV Couple/moment :→

→ Externally applied moment at a point on the beam is taken as couple or moment load on the beam.

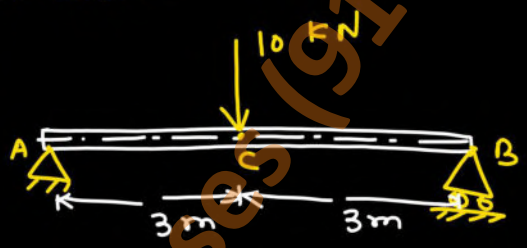
example



F.B.D



Q Find the support reactions for fig. shown below.



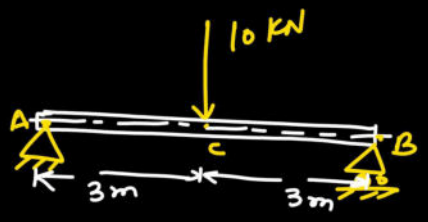
Solution :→

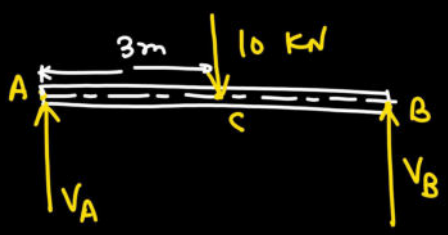
∴ Algebraic summation all horizontal, vertical and moment about any point must be zero

$$\Sigma F_x = 0 = \Sigma F_H$$

$$\Sigma F_y = \Sigma F_V = 0$$

$$\Sigma M = 0$$





$\because \Sigma F_H = 0$

$\boxed{0 = 0}$

Again $\Sigma F_V = 0$

$+V_A + V_B - 10\,KN = 0$

$\boxed{V_A + V_B = 10\,KN}$ —— (1)

Again let total moment about 'A' is zero

$\Sigma M_A = 0$

$+10\,KN \times 3\,m - V_B \times 6\,m = 0$

$30\,KN\text{-}m = V_B \times 6\,m$

$\Rightarrow V_B = \dfrac{\cancel{30}^{5}\,KN\text{-}\cancel{m}}{\cancel{6\,m}}$

$\boxed{V_B = 5\,KN}$ Ans//

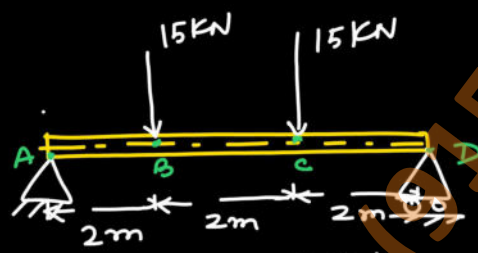
from eqn (1)

$V_A + V_B = 10\,KN$

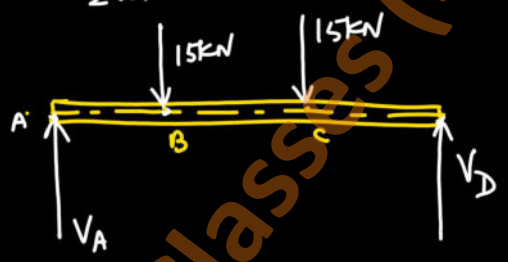
$V_A + 5\,KN = 10\,KN$

$\boxed{V_A = 5\,KN}$ Ans//

Q//



Sol^n:-



$\Sigma F_x = \Sigma F_H = 0$

$\boxed{0 = 0}$

$\because \Sigma F_y = \Sigma F_V = 0$

$+V_A - 15\,KN - 15\,KN + V_D = 0$

$\boxed{V_A + V_D = 30\,KN}$ —— (1)

let moment about A is zero

$\Sigma M_A = 0$

$+15\,KN \times 2\,m + 15\,KN \times 4\,m - V_D \times 6\,m = 0$

$\Rightarrow 30\,KN\text{-}m + 60\,KN\text{-}m = 6 \times V_D$

$\Rightarrow V_D = \dfrac{\cancel{90}^{15}\,KN\text{-}\cancel{m}}{\cancel{6\,m}} = 15\,KN$

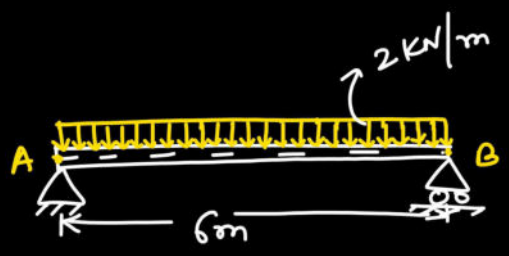
$\boxed{V_D = 15\,KN}$ Ans//

Again from equation (1) $\because V_A + V_D = 30\,KN$

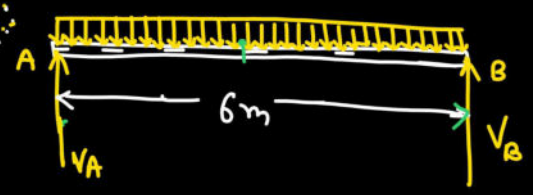
$V_A + 15\,KN = 30\,KN$

$\boxed{V_A = 15\,KN}$ Ans//

Q



Solution:



$\because \Sigma F_H = 0$

$\boxed{0 = 0}$

Again $\Sigma F_V = 0$

$+V_A + V_B - \frac{2KN}{m} \times 6m = 0$

$\boxed{V_A + V_B = 12\ KN}$ — (1)

let total moment about point 'A' is zero.

$\Sigma M_A = 0$

$+ \frac{2KN}{m} \times 6m \times \frac{6m}{2} - 6 \times V_B = 0$

$36\ KN\text{-}m = 6m \times V_B$

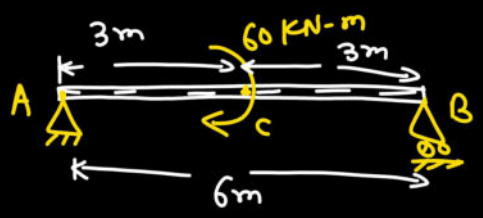
$V_B = \frac{36\ KN\text{-}m}{6m} = 6KN$ Ans//

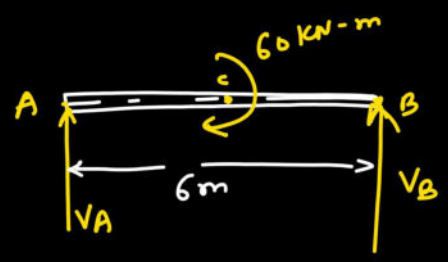
$\therefore$ from equation (1) $V_A + V_B = 12KN$

$V_A + 6KN = 12KN$, $\boxed{V_A = 6KN}$ Ans//

Q A simply-supported beam of length 6m carry a couple of moment 60KN-m at mid span. The support reactions will be

Sol^n :→





$\because \Sigma F_H = 0$

$\boxed{0 = 0}$

Again $\Sigma F_V = 0$

$V_A + V_B = 0$

$\boxed{V_A = -V_B}$ — (1)

$\because$ let moment about point 'A' $= 0$

$\Sigma M_A = 0$

$+60\ KN\text{-}m - 6 \times V_B = 0$

$V_B = \dfrac{60\ KN\text{-}m}{6\ m} = 10\ KN$

$\boxed{V_B = 10\ KN}$ Ans (i)

from equation (1)

$\boxed{V_A = -V_B = -10\ KN}$ Ans (ii)

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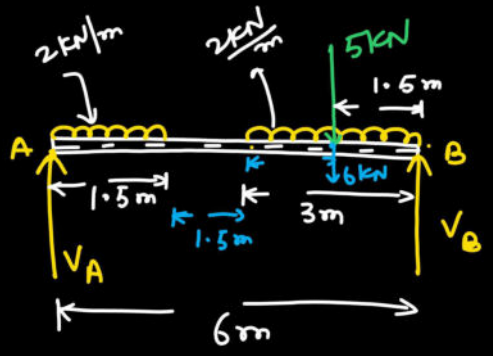
Q/ Find reactions at the supports.

Solution:-



$\because \Sigma F_H = 0$

$\boxed{0 = 0}$

$\because$ Again $\Sigma F_V = 0$

$+V_A - 2\frac{KN}{m} \times 1.5\,m - 2\frac{KN}{m} \times 3\,m - 5KN + V_B = 0$

$\Rightarrow \boxed{V_A + V_B = 14\ KN}$ — (1)

let $\Sigma M_A = 0$

$$+ 1.5\,m \times \frac{2\,KN}{m} \times \frac{1.5}{2}\,m + \frac{2\,KN}{m} \times 3\,m\left(3\,m + \frac{3\,m}{2}\right) + 5\,KN \times 4.5\,m - V_B \times 6\,m = 0$$

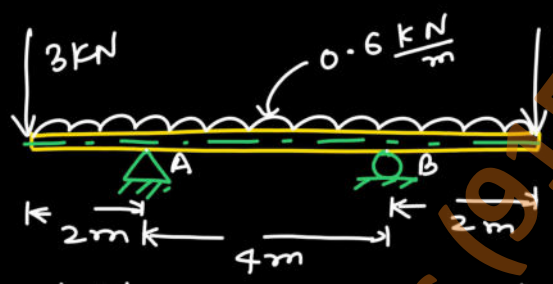
$$\Rightarrow V_B = \frac{51.75\,KN\text{-}m}{6\,m}$$

$$\boxed{V_B = 8.625\,KN}$$ Ans//

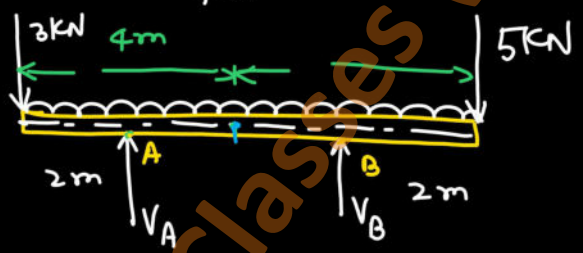
from equation ① $V_A + V_B = 14\,KN$

$V_A + 8.625\,KN = 14\,KN$, $\Rightarrow \boxed{V_A = 5.375\,KN}$ Ans//

Q//



Sol^n:→



$\because \Sigma F_H = 0$

$\boxed{0 = 0}$

$\because \Sigma F_V = 0$

$-3\,KN - 0.6\,\frac{KN}{m} \times 8\,m + V_A + V_B - 5\,KN = 0$

$\Rightarrow \boxed{V_A + V_B = 12.8\,KN}$ —— ①

$\because \Sigma M_A = 0$

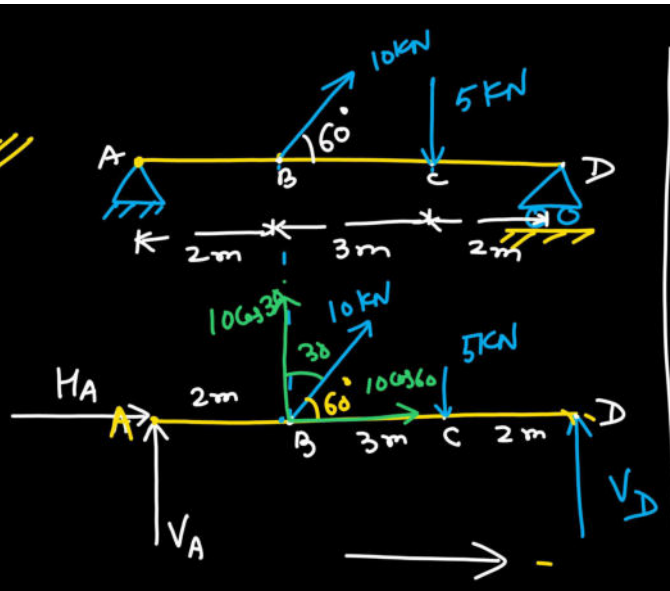
$$-3\,KN \times 2\,m + 0.6\,\frac{KN}{m} \times 8\,m \times 2\,m - V_B \times 4\,m + 5\,KN \times 6\,m = 0$$

$$\Rightarrow V_B = \frac{33.6\,KN\text{-}m}{4\,m}$$

$$\boxed{V_B = 8.4\,KN}$$ Ans//

∴ from equation ① $V_A + V_B = 12.8\,KN$, $\Rightarrow V_A + 8.4\,KN = 12.8\,KN$, $\boxed{V_A = 4.4\,KN}$ Ans//

Q//



$\Sigma F_H = 0$

$+H_A + 10\cos 60 = 0$

$H_A = -10\cos 60$

$\boxed{H_A = -5KN}$ Ans//

∵ Again

$\Sigma F_V = 0$

$+V_A + 10\cos 30 - 5KN + V_D = 0$

$\Rightarrow V_A + V_D = 5 - 10\cos 30$

$\Rightarrow \boxed{V_A + V_D = -3.66\ KN}$ — (1)

Total moment about point 'A' = 0

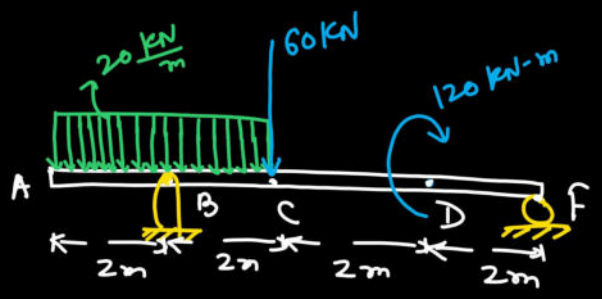
$\Sigma M_A = 0$

$-10\cos 30 \times 2m + 5KN \times 5m - V_D \times 7m = 0$

$V_D = \frac{7.675\ KN\cdot m}{7m} = 1.097\ KN$ Ans//

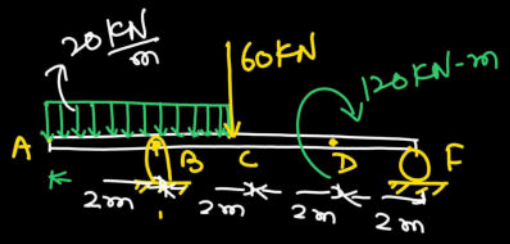
from equation (1) $V_A + V_D = -3.66KN$, $V_A = -3.66 - 1.097 = -4.75\ KN$ Ans//

Q// Determine the support reaction of the beam loaded as shown in fig.

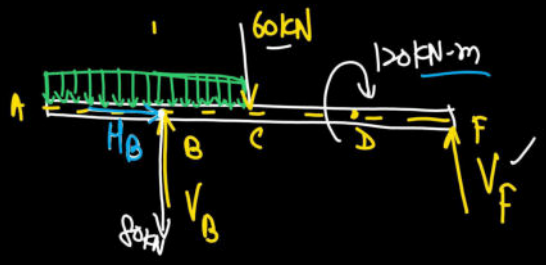


Engg. Mechanics

Q



Sol^n:



$\because \Sigma F_H = 0$

$\boxed{H_B = 0}$ Ans,

Again $\Sigma F_V = 0$

$-20\,\frac{KN}{m} \times 4\,m + V_B - 60\,KN + V_F = 0$

$\Rightarrow \boxed{V_B + V_F = 140\,KN} \rightarrow (1)$

let total moment about point 'B' $= 0$

$\Sigma M_B = 0$

$+60\,KN \times 2\,m + 120\,KN\text{-}m - 6V_F = 0$

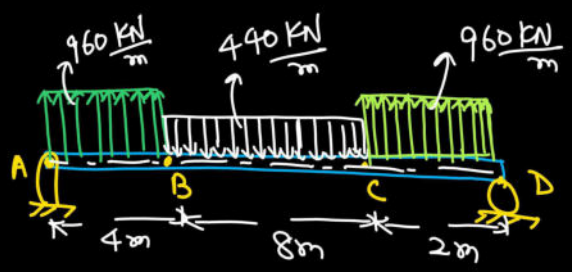
$V_F = \frac{240\,KN\text{-}m}{6\,m}$

$\boxed{V_F = 40\,KN}$ Ans//

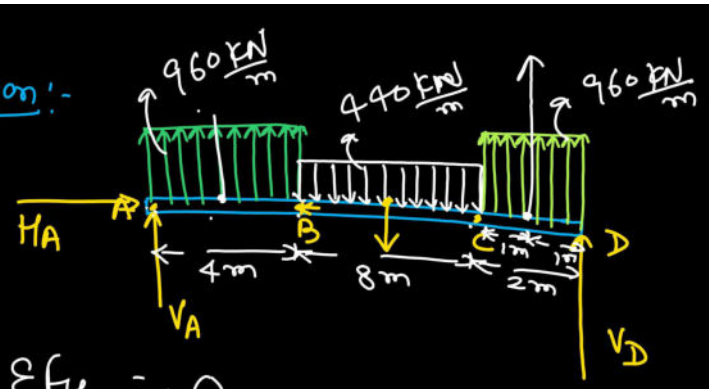
from eqn (1)

$V_B + V_F = 140$, $V_B + 40 = 140$, $\boxed{V_B = 100}$

Q Determine the supports reactions of the beam loaded as shown in fig:-



Solution:-



$\because \Sigma F_H = 0$

$+\boxed{H_A = 0}$ Ans//

Again $\Sigma F_V = 0$

$$+V_A + 960\frac{KN}{m} \times 4m - 440\frac{KN}{m} \times 8m + 960\frac{KN}{m} \times 2m + V_D = 0$$

$\Rightarrow V_A + V_D + 2240 = 0$

$\Rightarrow \boxed{V_A + V_D = -2240}$ — (1)

Again let total moment about point A = 0

i.e $\Sigma M_A = 0$

$$-960\frac{KN}{m} \times 4m \times 2m + 440\frac{KN}{m} \times 8m \times 8m - 960\frac{KN}{m} \times 2m \times 13m - V_D \times 14 = 0$$

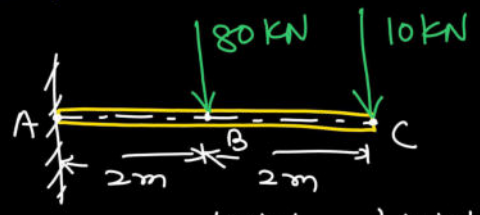
$$V_D = \frac{-4480}{14} = -320\ KN \text{ Ans//}$$

$\therefore$ from eqn (1)

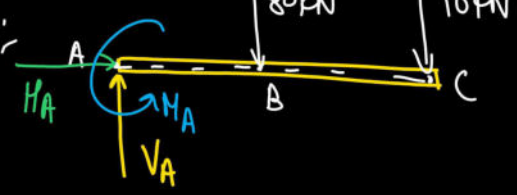
$V_A + V_D = -2240$, $V_A - 320 = -2240$

$\Rightarrow \boxed{V_A = -1920\ KN}$ Ans//

Q. Find the supports reactions of beam shown below:-



Solution:



$\because \Sigma F_H = 0$

$\boxed{H_A = 0}$ Ans//

$\because \Sigma F_V = 0$

$+V_A - 80\,KN - 10\,KN = 0$

$\Rightarrow \boxed{V_A = 90\,KN}$

Ans//

let total moment about point 'A' = 0

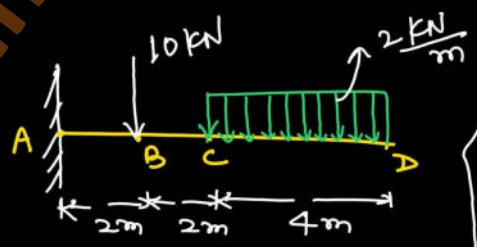
$\Sigma M_A = 0$

$-M_A + 80\,KN \times 2\,m + 10\,KN \times 4\,m = 0$

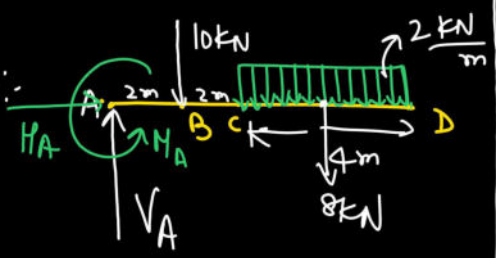
$M_A = 160\,KN\text{-}m + 40\,KN\text{-}m$

$\boxed{M_A = 200\,KN\text{-}m}$ Ans//

Q.



Solution:



$\because \Sigma F_H = 0$

$\boxed{H_A = 0}$ Ans//

$\because \Sigma F_V = 0$

$+V_A - 10\,KN - \frac{2\,KN}{m} \times 4\,m = 0$

$\boxed{V_A = 18\,KN}$

Ans//

let total moment about point 'A' = 0

$$\Sigma M_A = 0$$

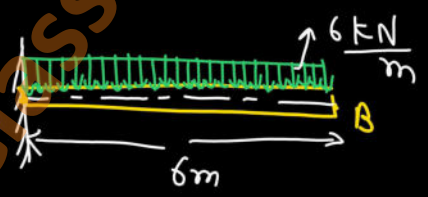
$$-M_A + 10\,KN \times 2\,m + 2\,\frac{KN}{m} \times 4\,m \times 6\,m = 0$$

$$\boxed{M_A = 68\,KN\text{-}m}$$ Ans//

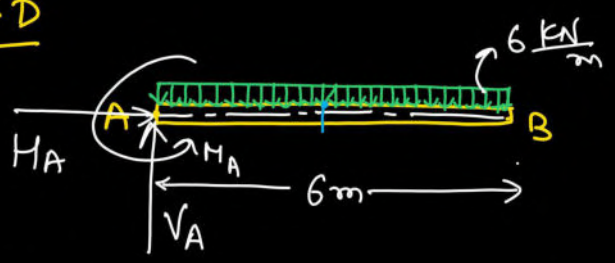
Engg. Mechanics

Q:- Find the support reactions of a Cantilever beam of span 6m carrying a UDL of $6\,\frac{KN}{m}$.

Solution:-



F.B.D



$\because \Sigma F_H = 0$

$\therefore$ $\boxed{H_A = 0}$ Ans//

∵ Again $\Sigma F_V = 0$

$$+V_A - \frac{6\,KN}{m} \times 6m = 0$$

$$\Rightarrow \boxed{V_A = 36\,KN} \text{ Ans//}$$

let total moment about point 'A' = 0

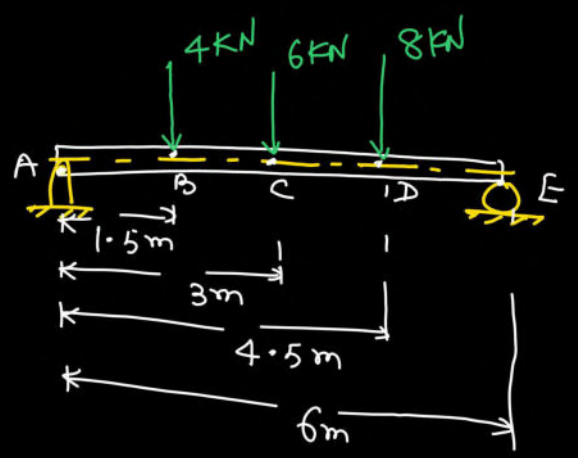
$$\Sigma M_A = 0$$

$$-M_A + \frac{6\,KN}{m} \times 6m \times 3m = 0$$

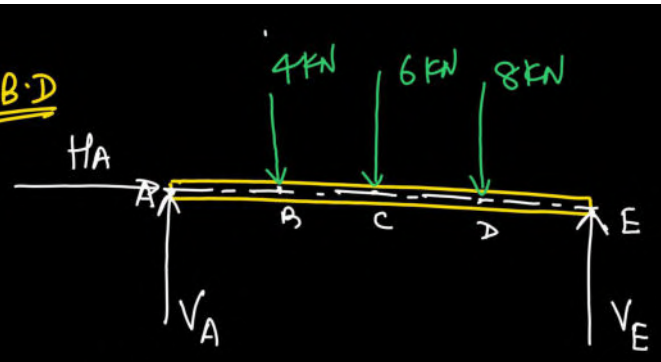
$$\boxed{M_A = 108\,KN\text{-}m} \text{ Ans//}$$

Q// A beam of 6m in span is simply supported with roller and hinge support at its ends. it carries three point load 4 KN, 6KN, and 8KN at 1.5 m, 3m and 4.5 m respectivelly from left support in vertical direction. calculate the support reactions.

Solution:



F.B.D



$\because \quad \Sigma F_H = 0$

$\boxed{H_A = 0}$ Ans//

$\because$ Again $\Sigma F_V = 0$

$+V_A - 4KN - 6KN - 8KN + V_E = 0$

$\boxed{V_A + V_E = 18\,KN}$ —— (1)

Again let total moment about point 'A' = 0

$\Sigma M_A = 0$

$+4KN \times 1.5m + 6KN \times 3m + 8KN \times 4.5 - V_E \times 6m = 0$

$\therefore \quad V_E = \frac{60\,KN\text{-}m}{6\,m} = 10\,KN$

$\therefore \boxed{V_E = 10\,KN}$ Ans//

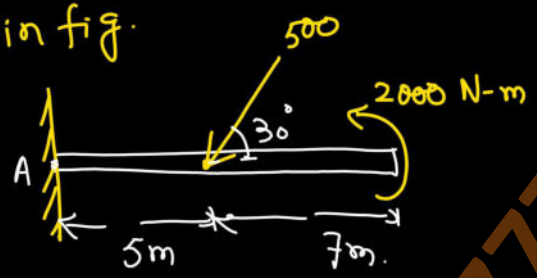
from equation (1)

$\because \quad V_A + V_E = 18\,KN$

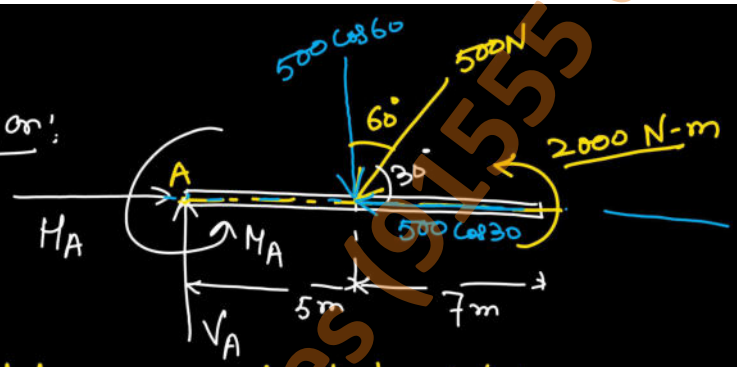
$V_A + 10\,KN = 18\,KN$

$\boxed{V_A = 8\,KN}$ Ans//

Q. Determine the moment about point A of the 500 N force and 2000 N-m Couple acting on a Cantilever beam as Shown in fig.



Solution:



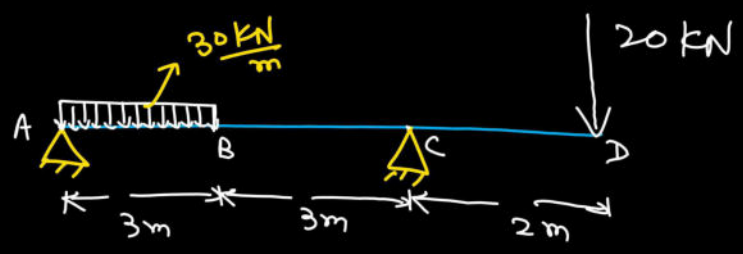
let total moment about point 'A' = 0

$$\Sigma M_A = 0$$

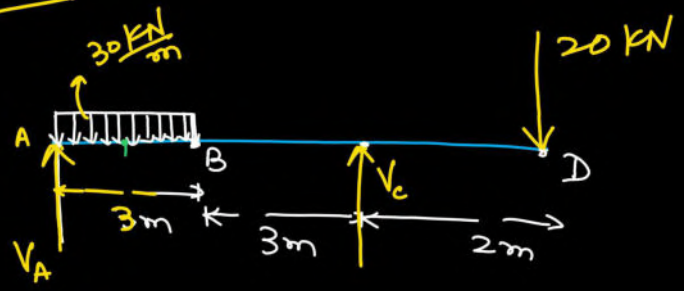
$$\therefore \quad -M_A + 500\cos 60 \times 5m - 2000\ N\text{-}m = 0$$

$$\boxed{M_A = -750\ N\text{-}m}$$ Ans

Q. for the beam as shown in fig Calculate the reaction at both supports



Solution: F.B.D



$\because \Sigma F_H = 0$

$\therefore 0 = 0$

Again $\because \Sigma F_V = 0$

$+V_A - 30\frac{KN}{m} \times 3m + V_c - 20KN = 0$

$V_A + V_c = 110\ KN$ — (1)

$\therefore$ Again let total moment about point 'A' $= 0$

$\Sigma M_A = 0$

$+30\frac{KN}{m} \times 3m \times 1.5m - V_c \times 6m + 20KN \times 8m = 0$

$V_c = \frac{295\ KN\text{-}m}{6\ m} = 49.167\ KN$

$V_c = 49.167\ KN$ Ans//

from equation (1)

$\because V_A + V_c = 110\ KN$

$V_A + 49.167\ KN = 110\ KN$

$V_A = 60.833\ KN$ Ans//

Determinate and Indeterminate Beams :→

We have three Equation of Equilibrium in plane :-

$$\Sigma F_H = 0 \text{ ——— (I)}$$

$$\Sigma F_V = 0 \text{ ——— (II)}$$

$$\Sigma M_R = 0 \text{ ——— (III)}$$

→ Determinate beams :→

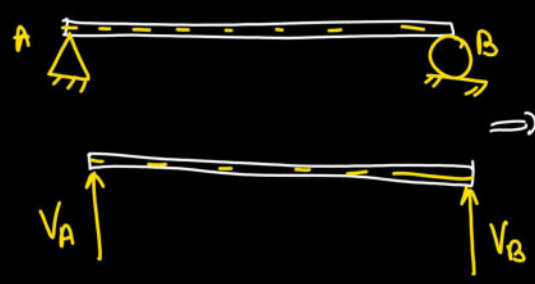
Those beams which have less than equal to three support reactions are called determinate beam.

example :— Simply supported, overhang Beam, Cantilever beam.

if $\boxed{n \leq 3}$ then Beam Determinate Beam

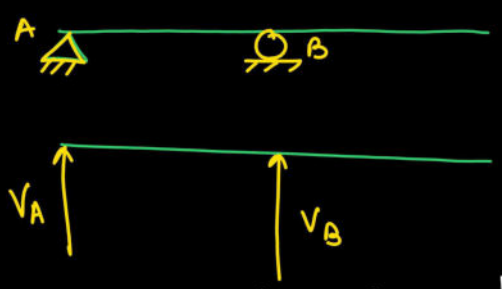
where n = number of support reaction.

example :



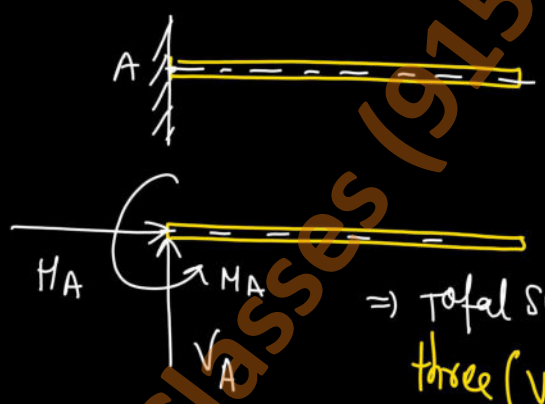
⇒ Total number of support reaction $= 2(V_A, V_B)$

overhang-Beam:-



→ In overhang Beam, total support reaction are two, therefore its Determinate Beam.

example cantilever:



⇒ total support reactions are three (V_A, H_A, M_A)

∴ Cantilever determinate beam.

Indeterminate:→

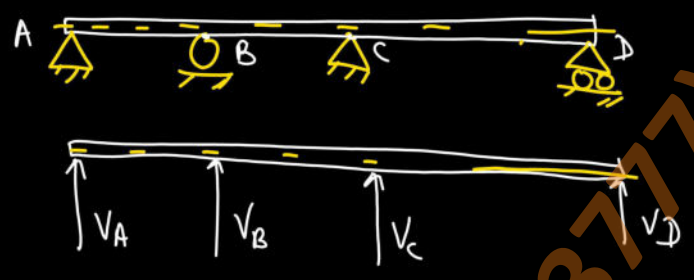
→ Those beams which have more than three supports reactions are called indeterminate beam.

i.e $\boxed{n > 3}$ → indeterminate beam

where n = number of support reactions.

example:- Continuous beam, propped cantilever, fixed beam.

example:- continuous beam:

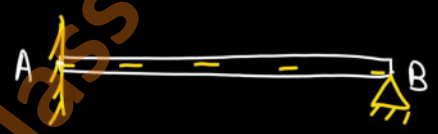


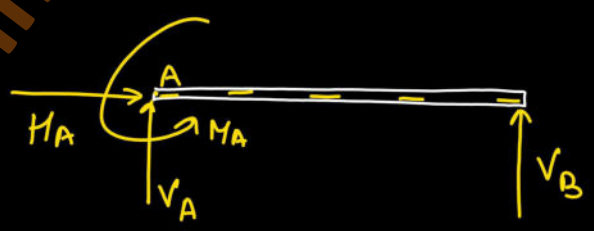
in above continuous beam,

number of support reaction (n) = 4

∴ It is indeterminate beam.

example propped cantilever:→



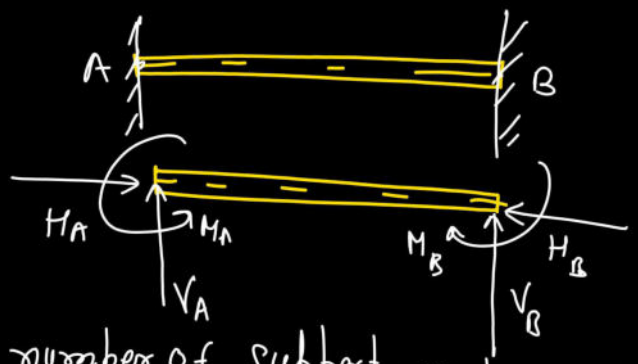


Total number of support reaction (n) = 4

(V_A, H_A, M_A, V_B)

∴ propped cantilever is an indeterminate beam.

example:- Fixed Beam:-

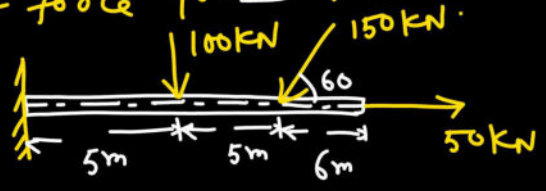


Total number of support reactions (n) = 6 (V_A, H_A, M_A, V_B, H_B, M_B)

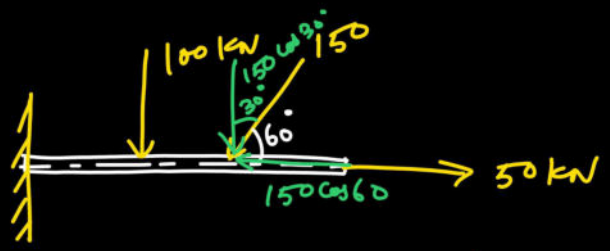
∴ fixed beam is an indeterminate beam.

Engg. Mechanics

Q. Determine the resultant force acting on the cantilever beam as shown in fig. and also find distance of resultant force from fixed end of beam.



Solution:-



$\because \Sigma F_x = -150\cos 60 + 50\,KN$

$= -75 + 50\,KN = -25\,KN$ ✓

$\Sigma F_y = -100 - 150\cos 30$

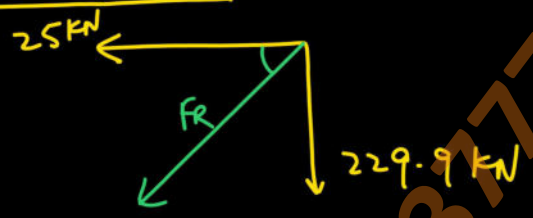
$= -229.90\,KN$ ✓

$\therefore$ Resultant force 'F_R' $= \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

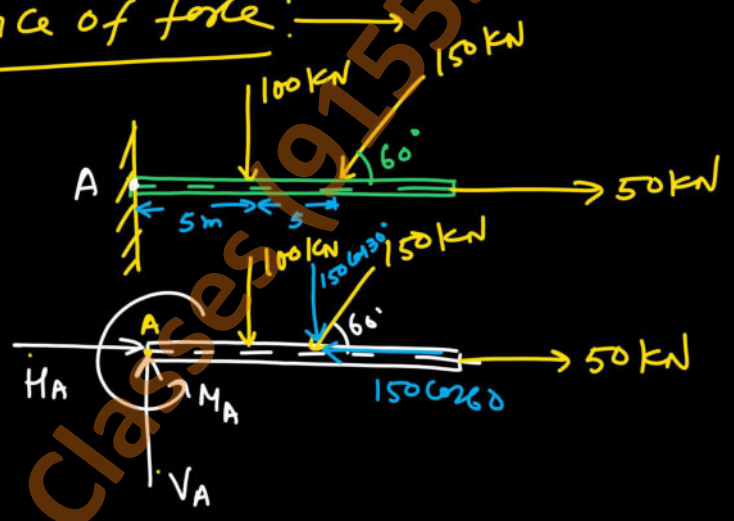
$= \sqrt{(-25\,kN)^2 + (229.90\,kN)^2}$

$= 231.26\ kN$ Ans//

Direction of Resultant force (F_R) :→



Distance of force :→



$\therefore \Sigma F_H = 0$

$+H_A - 150\cos 60 + 50\,kN = 0$

$H_A - 75 + 50 = 0$

$\boxed{H_A = 25\,kN}$

$\because \Sigma F_y = 0$

$+V_A - 100 - 150\cos 30 = 0$

$V_A = 100 + 150\cos 30 = 229.90\,kN.$

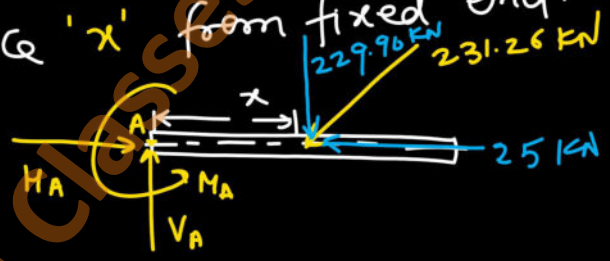
let total moment about point A = 0

$$\Sigma M_A = 0$$

$$-M_A + 100\,kN \times 5\,m + 150\,(\cos 30) \times 10\,m = 0$$

$$\boxed{M_A = 1799.04\ kN\text{-}m}$$

let resultant force (F_R) will act at a distance 'x' from fixed end.



∴ cantilever will remain in equilibrium

then $\Sigma M_A = 0$

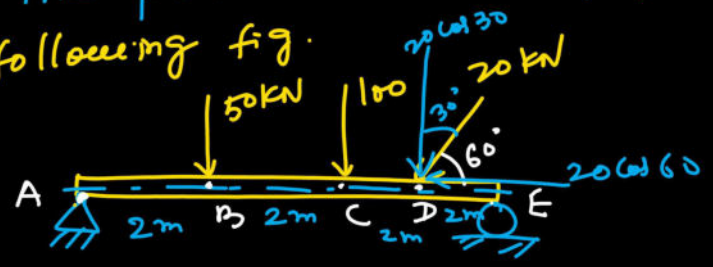
$$-M_A + 229.90x = 0$$

$$\Rightarrow 229.9x = M_A = 1799.04\ kN\text{-}m$$

$$\Rightarrow x = \frac{1799.04\ kN\text{-}m}{229.9\ kN} = 7.825\ m$$

$$\boxed{x = 7.825\,m}$$ Ans

Q. Find the position of resultant force in following fig.



$\Sigma F_x = -20\cos 60 = -10$ KN

$\Sigma F_y = -50 - 100 - 20\cos 30$

$= -167.32$ KN

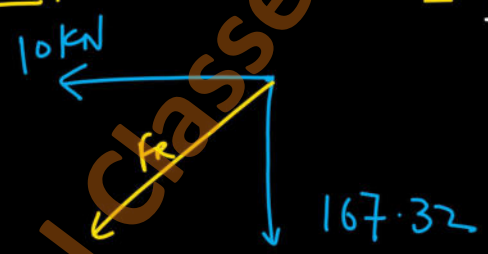
$\therefore$ Resultant force $(F_R) = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

$= \sqrt{(-10\text{ KN})^2 + (-167.32)^2}$

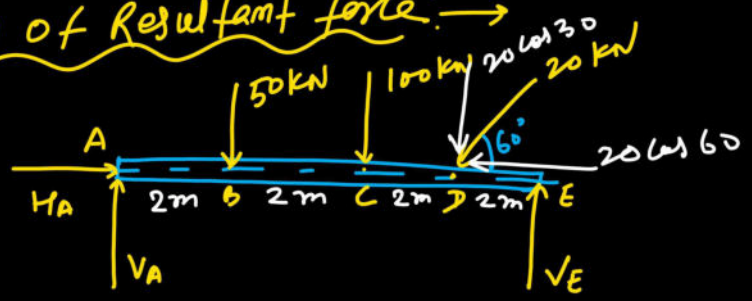
$= \underline{167.62 \text{ KN}}$

Ans//

Direction:-



Position of Resultant force:→



Eqn of eqbm :-

$\Sigma F_H = 0$

$+H_A - 20\cos 60 = 0$

$\boxed{H_A = 10 \text{ KN}}$

Again $\Sigma F_V = 0$

$+V_A - 50 - 100 - 20\cos 30 + V_E = 0$

$$\boxed{V_A + V_E = 167.32\ kN} \quad (1)$$

Again let $\Sigma M_A = 0$

$+50kN \times 2m + 100kN \times 4m + 20\cos 30 \times 6m - V_E \times 8 = 0$

$$V_E = \frac{603.92\ kN\text{-}m}{8m} = 75.49\ kN.$$

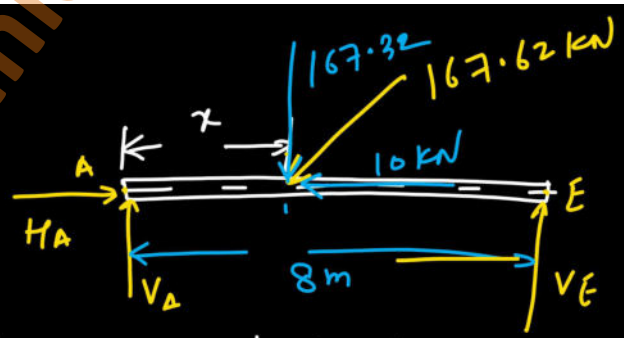
from eqn (1)

$\because V_A + V_E = 167.32\ kN$

$V_A + 75.49 = 167.32\ kN$

$V_A = 167.32 - 75.49\ kN$

$$\boxed{V_A = 91.83\ kN}$$



let resultant force ($F_R = 167.62kN$) will act at a distance 'x' from left end of beam

let $\Sigma M_A = 0$

$+167.32 \times x - V_E \times 8m = 0$

$$167.32x = V_E \times 8 = 75.49\,KN \times 8m$$

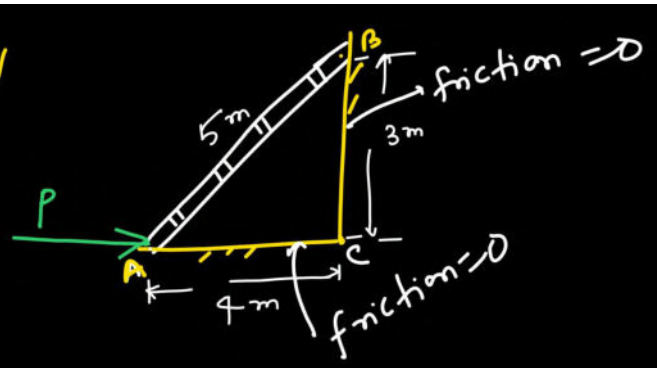
$$x = \frac{75.49 \times 8m}{167.32}$$

$$\boxed{x = 3.61\ m}$$ Ans/

Engg. Mechanics

Q A Ladder AB of length 5m and weight 600N is resting against a smooth wall, as shown in fig. Determine the force 'P' required to maintain the equilibrium of ladder.

weight(W) = 600N



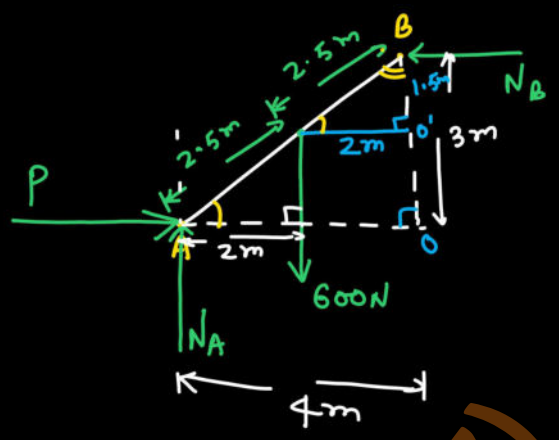
Solution:→

∵ ladder is in equilibrium

∴ $\Sigma F_H = 0$

$+P - N_B = 0$

∴ $\boxed{P = N_B}$ — (1)



Again ∵ $\Sigma F_V = 0$

$+N_A - 600 = 0$

$\boxed{N_A = 600 \text{ newton}}$

∴ let total moment about point A = 0

$\Sigma M_A = 0$

$+600N \times 2m - N_B \times 3m = 0$

$N_B = \frac{\cancel{600}^{200} N \times 2\cancel{m}}{\cancel{3m}} = 400 \text{ N}$

from eqn (1)

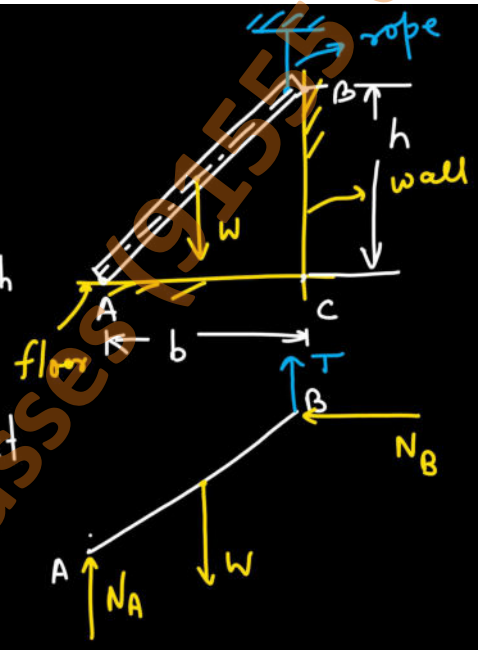
$N_B = P = 400 \text{ N}$ Ans//

Q. A heavy load 'W' leans against a frictionaless wall and rest on a frictionaless floor, for the equilibrium, the upper end is supported by a means of a rope as shown in fig, find reaction at vertical wall.

let L = length of load.

b = distance of contact of load with floor from wall.

h = height of contact point of load with wall.



let T = tension in the rope.

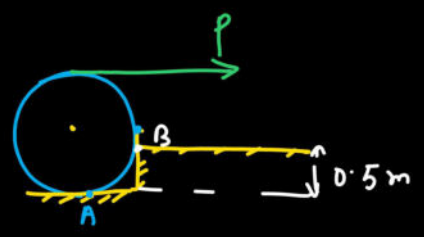
N_B = Reaction at vertical wall.

∵ load in eqbm :—

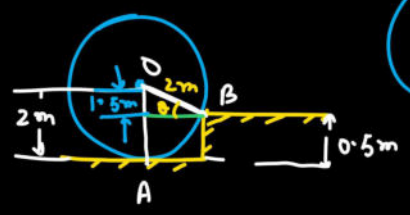
$$\Sigma F_H = 0$$

$$\boxed{N_B = 0}$$ Ans//

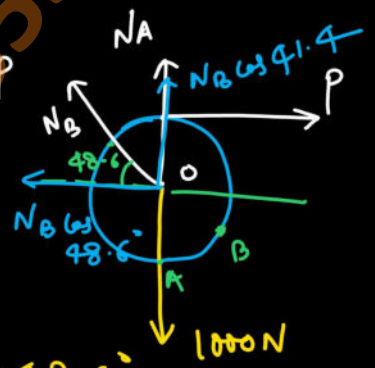
Q. Find the horizontal force P required to move the cylinder of weight 1000N and radius 2m as shown from ditch.



Solution :→



⇒



$\theta = 48.6°$

$\therefore \sin\theta = \frac{1.5\,m}{2\,m}$

$\therefore \sin\theta = 0.75$

$\boxed{\theta = 48.6°}$

→ when cylinder starts coming out of ditch contact at A vanishes, therefor $\boxed{N_A = 0}$

$\because \Sigma F_x = 0$

$-N_B \cos 48.6 + P = 0$

$\boxed{P = N_B \cos 48.6}$ — (1)

$\Sigma F_V = 0$

$N_B \cos 41.4 - 1000 = 0$

$N_B = \frac{1000 \text{ newton}}{\cos 41.4} = 1333.13 \text{ newton}$

from equation ①

$P = N_B \cos 48.6 = 1333.13 \times \cos 48.6$

$= \underline{881.62 \text{ N}}$ Ans //

Q ____ is statically indeterminate Beam

(Cantilever beam / Simply supported beam / Continuous beam)

Question 1-c →

Ans :- Continuous beam

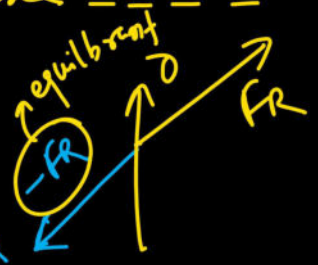
Q point load is applied over an area rather than at a single point.

(True / False)

Question 1-d →

Ans :→ False.

Q. The equilibrant is the force ----



(a) Causes motion ✗

(b) ✓ Balance the net force

(c) increases the acceleration.

(d) Acts perpendicular to the equilibrium.

Q. ----- is a horizontal structural member subjected to transverse load perpendicular to axis

(a) strut (b) column (c) ✓ Beam (d) Truss.

Q. For simply supported Beam, the moment at the support is always -----

(a) Maximum

(b) ✓ zero

(c) Minimum

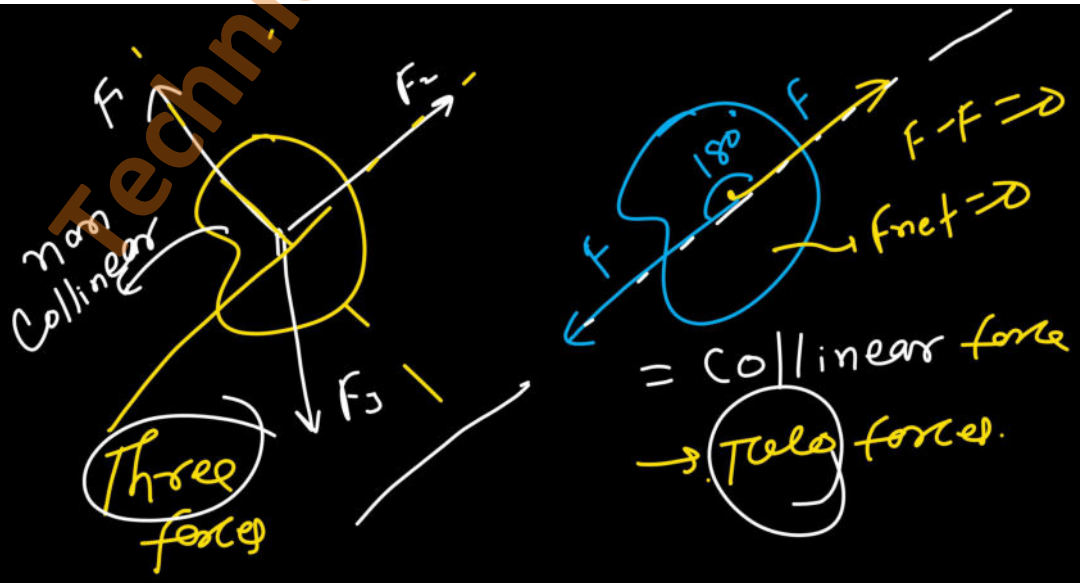
(d) None of these.

Q. ------ is a beam with one end fixed and other end simply-supported.

(a) fixed beam
(b) Continuous beam
(c) overhang beam
(d) propped cantilever.

Q. If the body is in equilibrium under the influence of set of non-collinear forces, then the minimum number of force has to be

(a) Two (b) Three (c) Four (d) five.



Engg. Mechanics

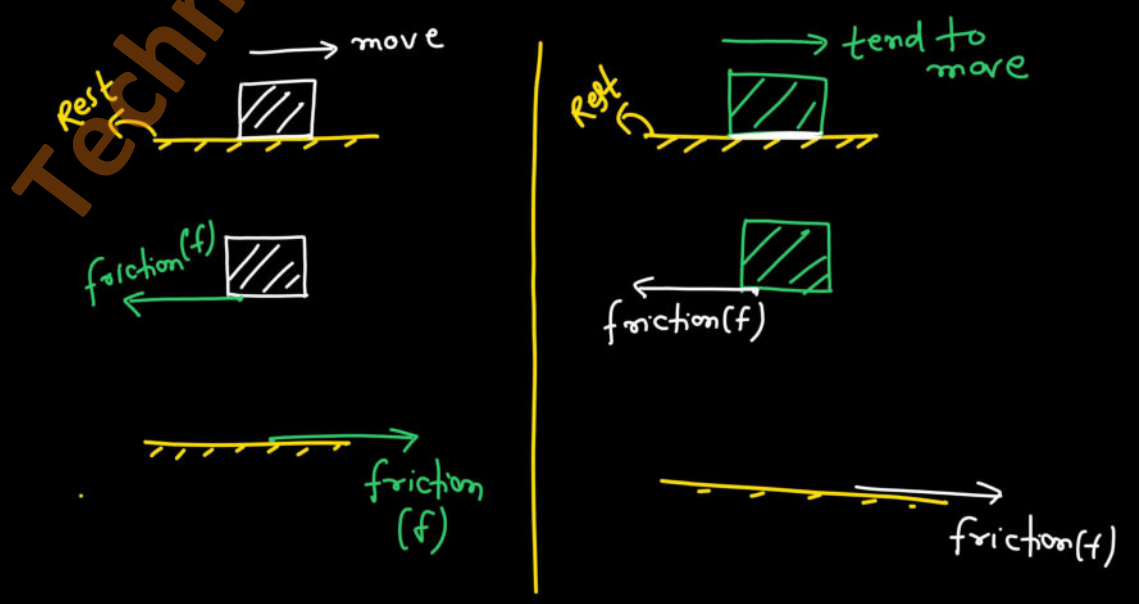
UNIT-03 : FRICTION

12 – 14 Marks
↳ 2 obj × 2 = 4 Marks

Q ↳ 6 Marks (Numerical)
↳ 4 Marks (Theory or numerical)

Friction or frictional force (f) :→

→ When one body moves (or tends to move) over the surface of another body, then a force is develope tangentially between the surfaces in contact is called frictional force or simply friction.

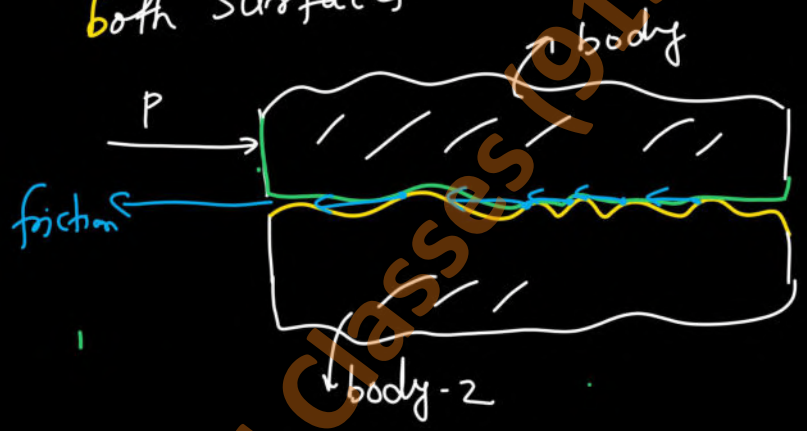


** → friction can support as well as oppose motion of the surface in contact.

** → But friction always oppose relative motion.

→ friction force is a contact type force.

** → friction force develope between surfaces in contact due nature (Roughness) of both surfaces.



Engg. Mechanics

Advantages of friction: →

followings are the important advantages of friction.

I vehicles moves on the roads, that are supported by friction.

II friction helps in applying breaks of the moving objects like, cycles, Bus etc.

III Gripping of an object is also due to friction.

(iv) pillings of objects over another objects are also supported by forction.

(v) friction is also use to produce Heat.

(vi) initially humans used to produce fire by the rubbing of one stone over other.

(vii) friction is also use to transform one form of energy into another form of energy.

Disadvantages of friction :→

followings are the some disadvantages of friction :→

(i) friction also do loss of energy in the form of heat.

(ii) friction also cause decrease in efficiency of the devices or machines.

(iii) friction also causes wier and tier of the objects.

(iv) friction also induces roughness of the surface of the objects.

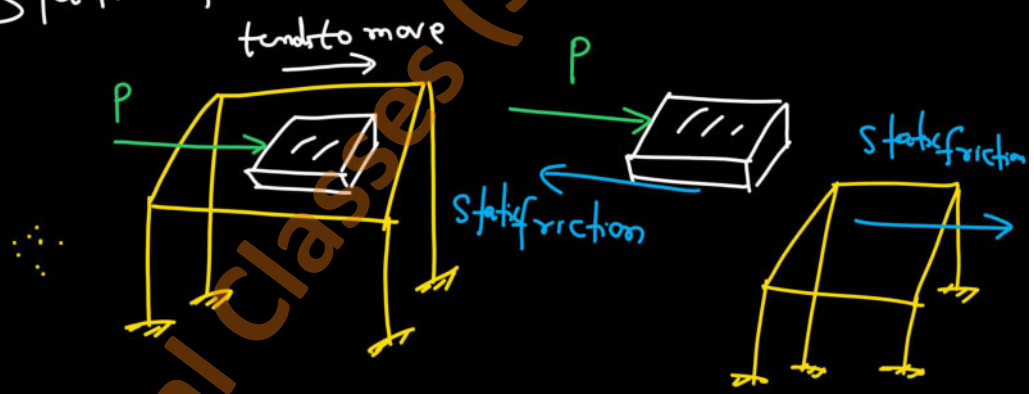
Types of friction :→

→ There are two types of friction :→

I Static friction II Dynamic friction or Kinetic friction.

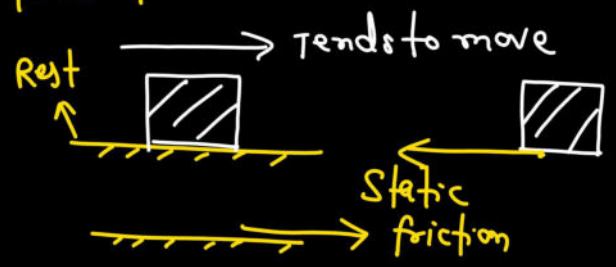
I Static friction :→

When one object is in the state of tend to move (but no moving) over the surface of another object then friction force develope at this condition between the surfaces in contact is called static friction.



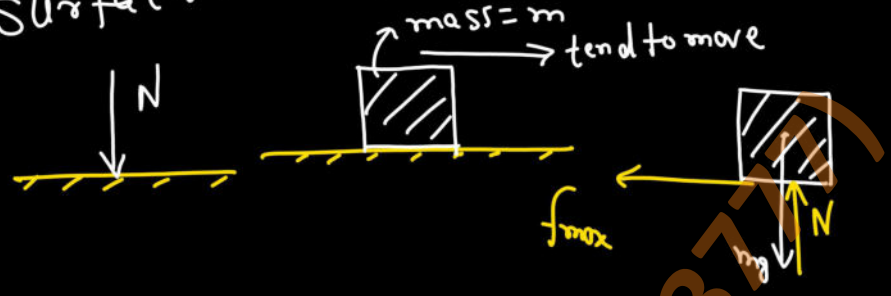
Laws of static friction :→

I. The direction of static friction on the object opposite to direction in which object tends to over the surface of another object



*****

II The maximum static friction is called limiting friction and it is directly proportional to Normal reaction between surfaces in contact



f_{max} → maximum static friction force or limiting friction force

$\therefore f_{max} \propto N$

or ** $\boxed{f_{max} = \mu_s N}$

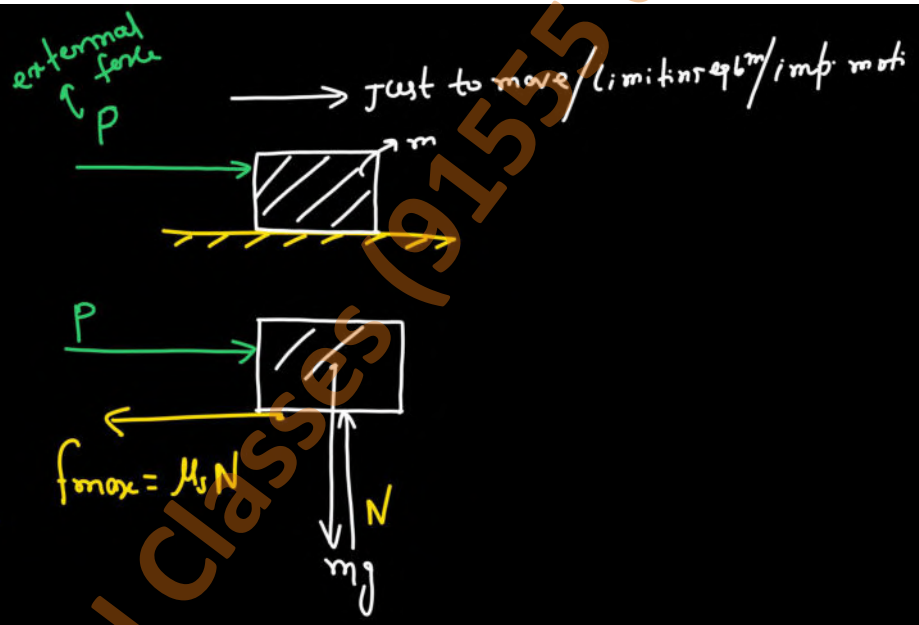
** μ_s → Coefficient of static friction.
→ It has no unit and no dimension.

→ value of coefficient of static friction (μ_s) lies between 0 and 1.

$$\boxed{0 < \mu_s < 1}$$

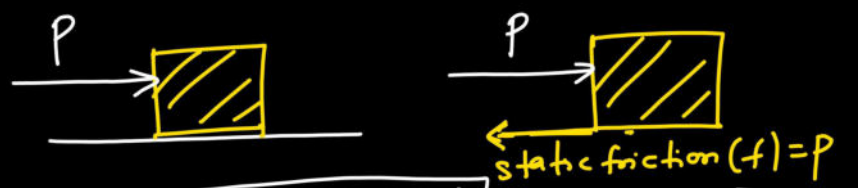
→ Its value depends on roughness, impurities (dust, oil, greece) between surfaces in contact

→ Max^m static friction force (Limiting friction force) will act only when an object is in the state of or condition of just to move / limiting eqb^m / on the verge of motion / impen-ding motion over the surface of another object.



$$\boxed{0 < \text{Static friction force } (f) \leq f_{max} = \mu_s N}$$

III If net external force on the object is less than maximum static friction force, then static friction force of magnitude equal to net external force and in opposite will act on the object



If $\boxed{P < f_{max} (\mu_s N)}$

$\therefore$ static friction force (f) = P

(iv) ** value of static friction force does not depend area of contact of surfaces.

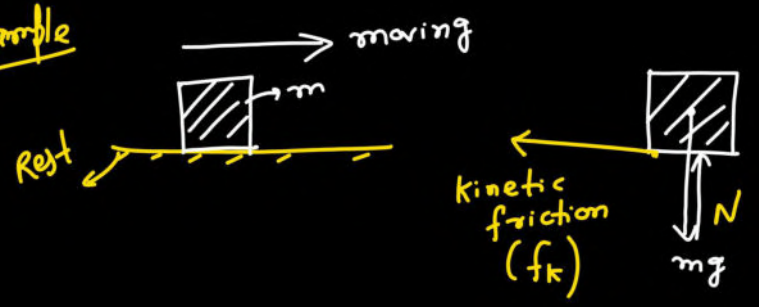
(v) Static friction force only depends on nature (roughness) of the surface in contact

Engg. Mechanics.

II Dynamic/Kinetic friction :→

→ when an object moves over the surface of another object, then, the friction force develope between surfaces in contact is called Dynamic or Kinetic friction.

example

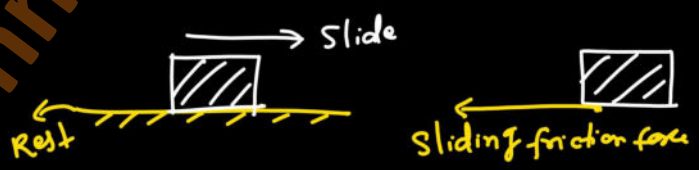


→ There are two types of Kinetic/Dynamic friction:—

(a) Sliding friction.

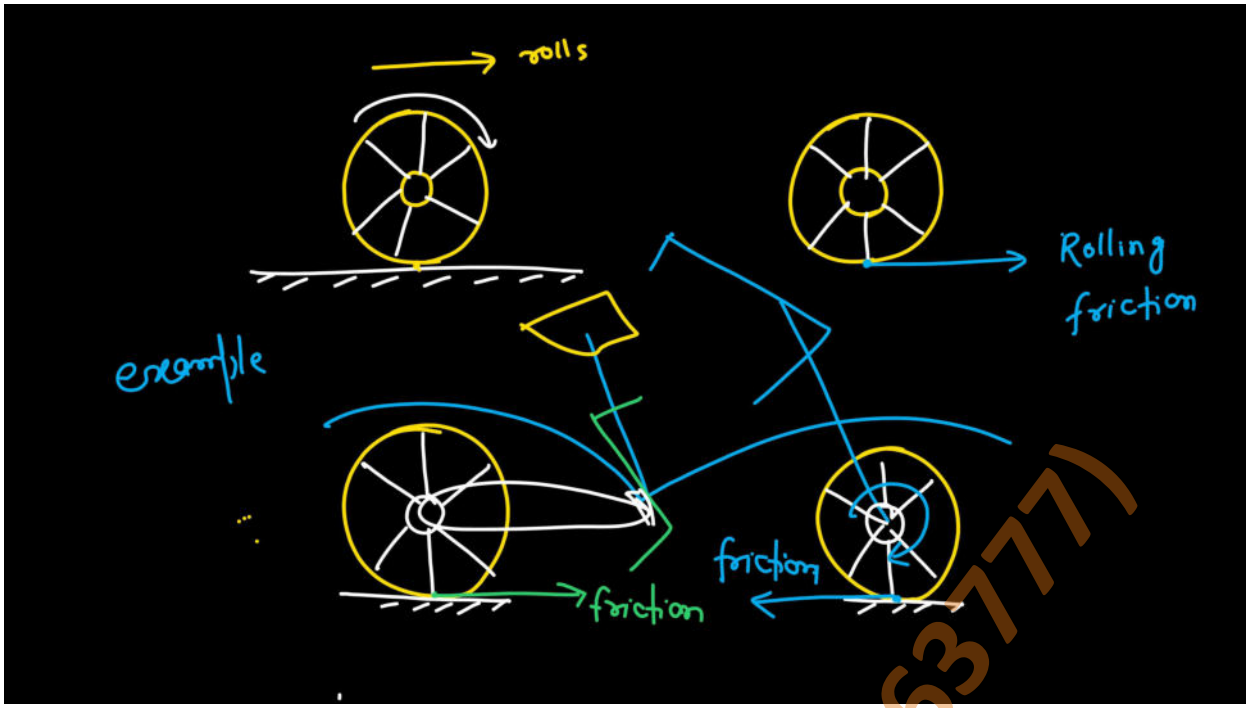
(b) Rolling friction.

(a) Sliding friction:→ When an object slides over the surface of another object, then the friction force between surfaces of two objects is called sliding friction.



(b) Rolling friction:→

→ When one object rolls over the surface of another object, then friction force between surfaces in contact is called rolling friction.



laws of kinetic friction :→

followings are the statements of laws of kinetic friction.

(i) Kinetic will always act in a direction opposite to direction of relative motion between two surfaces in contact

(ii) The value of kinetic friction is given by

$$\boxed{f_K = \mu_K N}$$

f_K → kinetic friction

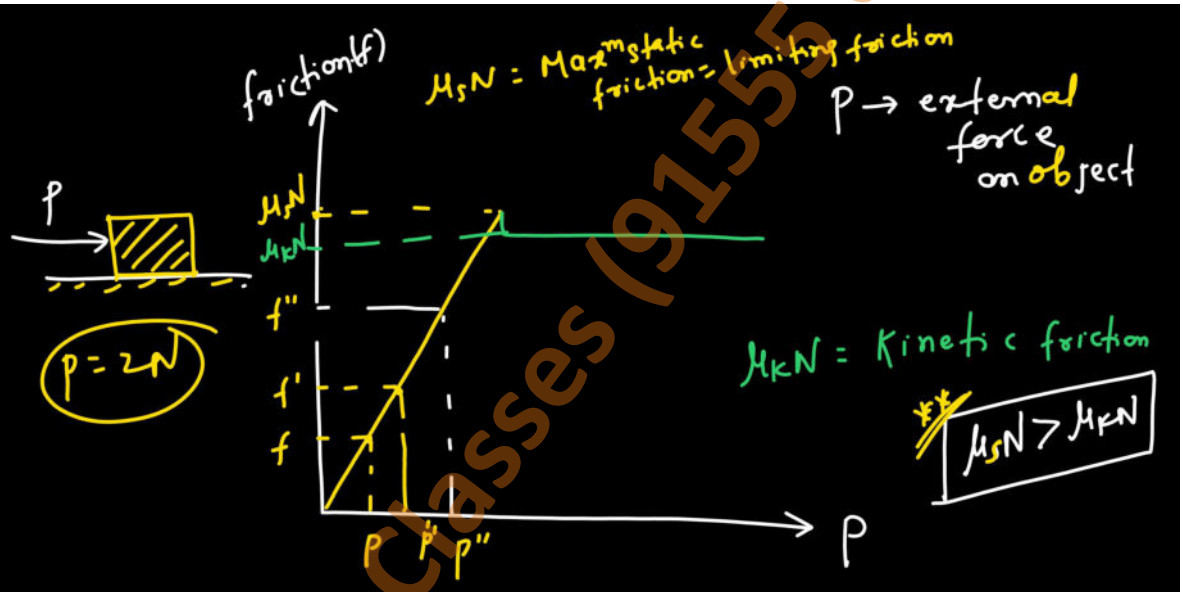
μ_K = Co-efficient of kinetic friction.

N = Normal reaction on the surface.

*** Note :→ value of co-efficient of static friction is always greater than value of co-efficient of kinetic friction. $\boxed{\mu_s > \mu_K}$

(iii) In generally value of Kinetic friction (f_k) is constant with respect to speed.

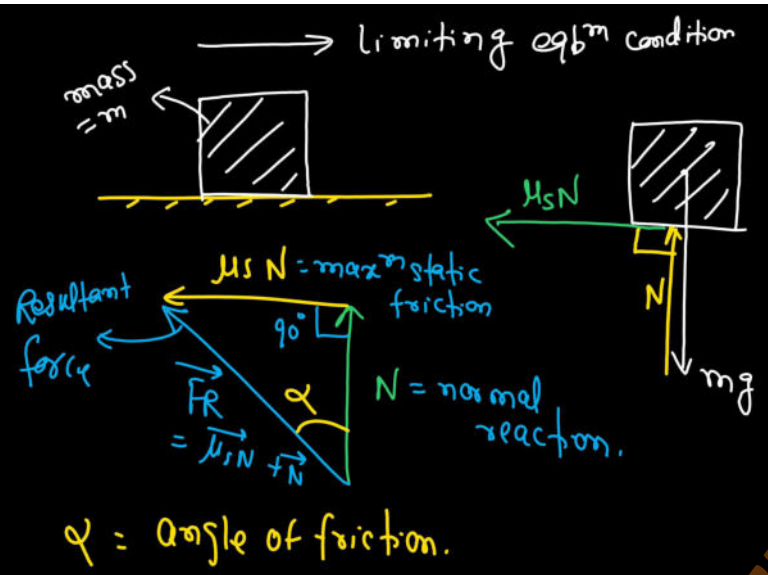
(iv) At Higher speed value of Kinetic friction increases with increase of speed of motion.



Engg. Mechanics

Angle of friction (α) :→

→ When an object is in limiting equilibrium on a horizontal surface, then the angle made by resultant force of normal reaction and maximum static friction with normal reaction is called angle of friction.



α = angle of friction.

from above right angle triangle

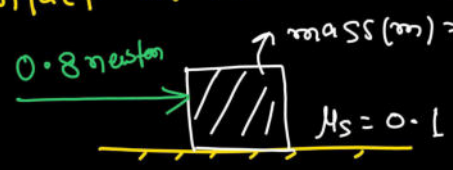
$$\tan\alpha = \frac{\mu_s \cancel{N}}{\cancel{N}} = \mu_s$$

**

$$\therefore \boxed{\text{Angle of friction}(\alpha) = \tan^{-1}(\mu_s)}$$

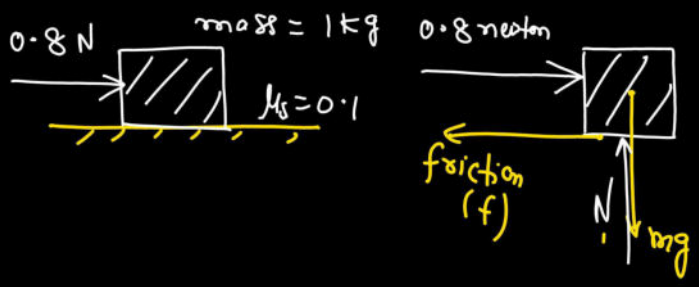
where μ_s = co-efficient of static friction.

Q Find the friction between surfaces in contact as shown in fig. –

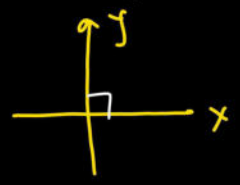


F.B.D

Solution:-



∵ Net/total perpendicular to horizontal surface must be zero.



$$\Sigma F_y = 0$$

$$N - mg = 0$$

$$N = mg = 1\,kg \times 9.81 = \underline{9.81 \text{ newton}}$$

∴ Maximum static or limiting friction force $(f_{max}) = \mu_s N$

$$\therefore f_{max} = 0.1 \times 9.81$$

$$= \underline{0.981 \text{ newton}}$$

∵ total external force $(0.8N) < f_{max}$

∴ object will be in <u>tend to motion</u> condition

∴ object is in eqbm in horizontal direction.

∴ Total/net force in horizontal dirn = 0

$$\Sigma F_H = 0$$

$$-f + 0.8 \text{ newton} = 0$$

$$\boxed{f = 0.8 \text{ newton}}$$ Ans//

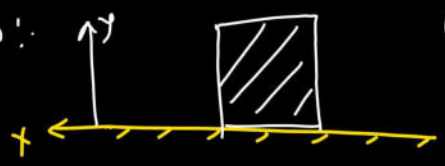
Q. A body of weight <u>200 N</u> lies on a <u>horizontal plane</u> for which co-efficient of friction is <u>0.4</u> Determine the followings

(a) limiting force of friction (v)

(b) Angle of friction (v)

(c) Horizontal force required to move. (a)

Solution:



Given
weight $(mg) = 200\,N$
$C.O.f\,(\mu_s) = 0.4$

(a) limiting force of friction (f_{max}) :—

$\because \Sigma F_y = 0$

$+N - mg = 0$

$\boxed{N = 200 \text{ newton}}$

$\therefore$ limiting friction force $(f_{max}) = \mu_s N$

$= 0.4 \times 200$ newton

$= 80$ newton Ans//

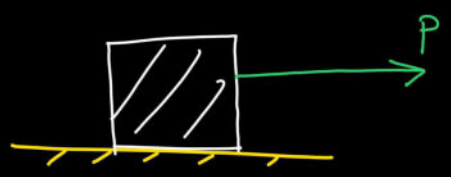
(b) Angle of friction (α) :→

$\alpha = \tan^{-1}(\mu_s)$

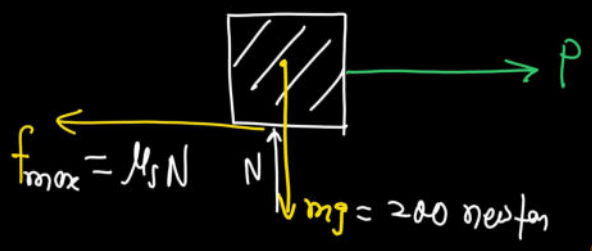
$= \tan^{-1}(0.4)$

$\boxed{\alpha = 21.80^\circ}$ Ans//

(c) Horizontal force to move :→



let a horizontal pull force 'P' is applied to move the body.



$\because \Sigma F_y = 0$

$+N - mg = 0$

$N = mg = 200$ newton

$\therefore$ To move the object in rightward direction

Total force in rightward direction	>	Total force in leftward direction

$P > f_{max}$

$\therefore P > \mu_s N$

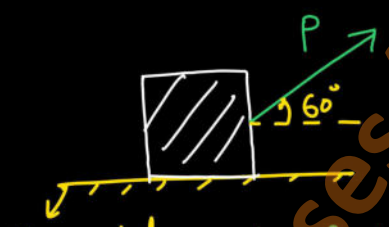
$P > 0.4 \times 200$ newton

$P > 80$ newton Ans/.

Q. A body of weight 300 N is lying on a rough horizontal plane having a co-efficient of friction between surfaces is 0.3. Find the magnitude of force which can move the body, while acting at an angle 60° with horizontal.

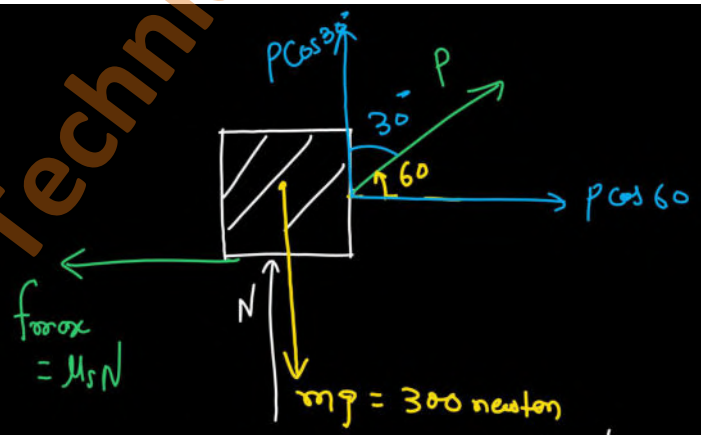
Solution :- Given weight (mg) = 300 newton

$\mu_s = 0.3$



Horizontal rough plane

let a force 'P' act at an angle 60° with horizontal can move the body.



∴ total force in vertical direction must be zero.

$\Sigma F_y = 0$

$+N - mg + P\cos 30 = 0$

$\therefore \quad N = mg - P\cos 30$

$$\boxed{N = 300 - P\cos 30}$$

Again to move the body in rightward direction:-

$\therefore \quad P\cos 60 > \mu_s N$

$\Rightarrow \quad P\cos 60 > 0.3(300 - P\cos 30)$

$\Rightarrow \quad P\cos 60 + 0.3P\cos 30 > 0.3 \times 300$

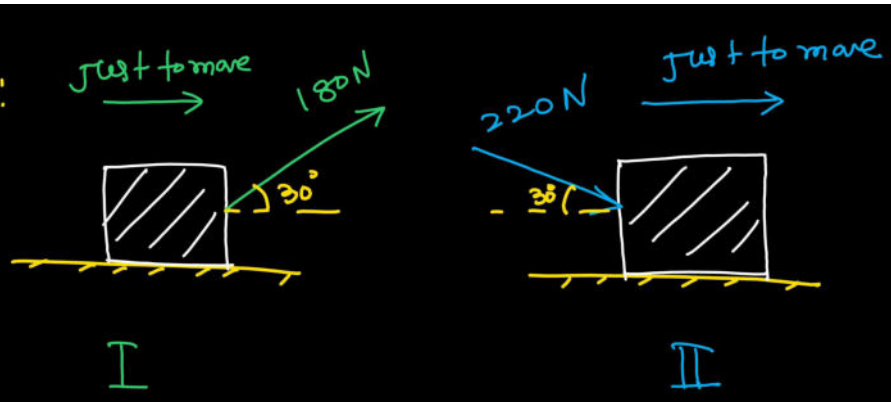
$P(\cos 60 + 0.3 \times \cos 30) > 0.3 \times 300$

$$\Rightarrow \quad P > \frac{0.3 \times 300}{\cos 60 + 0.3 \times \cos 30}$$

$$\boxed{P > 118.45 \text{ newton}}$$ Ans//

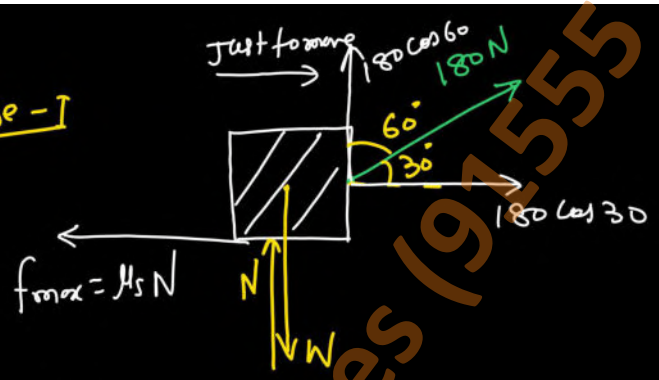
Q. A body resting on a rough horizontal plane required a pull of 180N inclined to 30° to the plane just to move, it was found that a push of 220N inclined at 30° to the plane just move the body. Determine the weight of the body and co-efficient of friction.

Solution:



let 'W' is the weight of the body and 'μ_s' is the co-efficient of friction between surfaces in contact

Case - I



$\therefore \quad \Sigma F_y = 0$

$+N - W + 180\cos 60 = 0$

$$\boxed{N = W - 180\cos 60}$$

$\therefore$ Body is in just to move condition that at rest and eqbm condn in horizontal dirn

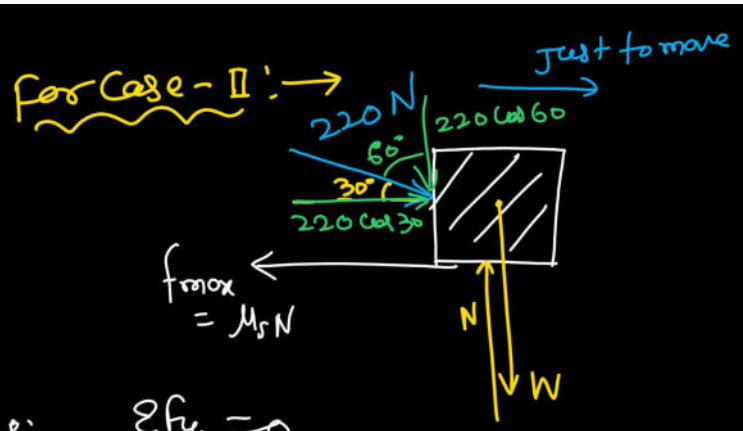
$\therefore \quad \Sigma F_H = 0$

$180\cos 30 - \mu_s N = 0$

$\Rightarrow \quad 180\cos 30 = \mu_s N$

$\Rightarrow \quad \boxed{180\cos 30 = \mu_s (W - 180\cos 60)} \quad (1)$

For Case - II :→



$\because \quad \Sigma F_y = 0$

$N - W - 220\cos 60 = 0$

$\boxed{N = W + 220\cos 60}$

$\because$ body is in limiting eqbm in horizontal dirn.

$\Sigma F_H = 0$

$+220\cos 30 - \mu_s N = 0$

$\Rightarrow \quad 220\cos 30 = \mu_s N$

$\Rightarrow \quad \boxed{220\cos 30 = \mu_s (W + 220\cos 60)} \quad \text{— (II)}$

$\therefore$ Dividing equation (I) by eqn (II) we get

$$\frac{\cancel{180}^{\,9}\ \cancel{\cos 30}}{\cancel{220}_{\,11}\ \cancel{\cos 30}} = \frac{\cancel{\mu_s}(W - 180\cos 60)}{\cancel{\mu_s}(W + 220\cos 60)}$$

$9(W + 220\cos 60) = 11(W - 180\cos 60)$

$9W + 9 \times \overset{110}{\cancel{220}} \times \frac{1}{\cancel{2}} = 11W - W \times \overset{90}{\cancel{180}} \times \frac{1}{\cancel{2}}$

$\Rightarrow \quad 9 \times 110 + 90 \times 11 = 11W - 9W$

$$\Rightarrow 2W = 9\times110 + 90\times11$$

$$W = \frac{\cancel{2}\times 9\times110}{\cancel{2}} = 990 \text{ newton}$$

$$\boxed{W = 990 \text{ newton}}$$ Ans//

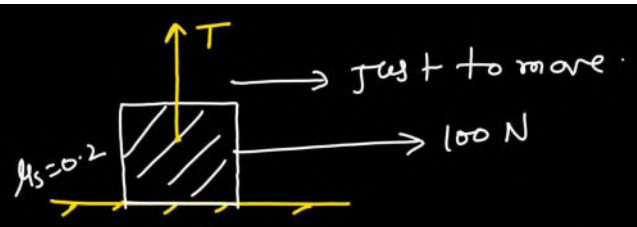
Again putting value of 'W' in equation (ii) we get

$$220\cos30 = \mu_s(990 + 220\cos60)$$

$$\Rightarrow \mu_s = \frac{220\cos30}{990 + 220\cos60}$$

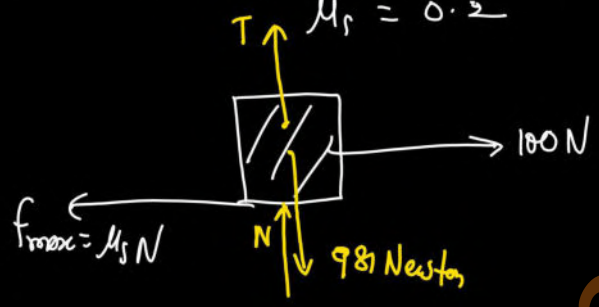
$$\boxed{\mu_s = 0.17}$$ Ans//

Q. A block of weight 981 N resting on a horizontal surface the coefficient of friction between surfaces is 0.2, a vertical cable attached to blocks provides partial support so that a man can pull the block horizontally with a force of 100 N, what will be the tension (T) in the cable, if the man is just able to move the block in rightward direction.



Solution:- Given Weight of block (mg) = 981 N

$\mu_s = 0.2$



$\because \Sigma F_y = 0$

$T + N - 981 \text{ newton} = 0$

$\boxed{N = 981 - T}$

$\because$ block is in Just to move condition in rightward direction.

$\therefore \Sigma F_H = 0$

$100 - \mu_s N = 0$

$100 = \mu_s N = 0.2(981 - T)$

$\Rightarrow 100 = 0.2(981 - T)$

$\Rightarrow 981 - T = \frac{100}{0.2} = \frac{1000}{2} = 500$

$\Rightarrow 981 - 500 = T$

$\Rightarrow \boxed{T = 481 \text{ newton}}$ Ans//

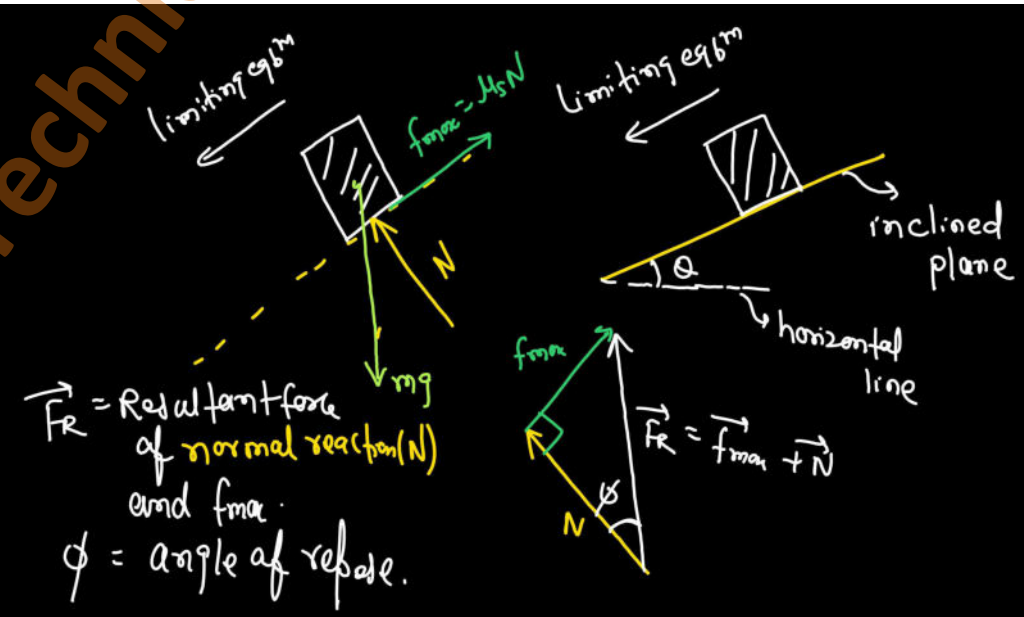
Note:- It is easier to pull an object than push the same object.

Engg. Mechanics

Friction on an inclined plane or surface :→

Angle of repose (ϕ) :→

→ When an object is in limiting equilibrium over an inclined plane, then angle made by resultant force (F_R) of normal reaction and maximum static friction force with normal reaction is called angle of repose.



$\vec{F_R}$ = Resultant force of normal reaction (N) and f_{max}.

ϕ = angle of repose.

in Right angle triangle:—

$$\tan\phi = \frac{f_{max}}{N} = \frac{\mu_s \cancel{N}}{\cancel{N}}$$

$$\tan\phi = \mu_s$$

$$\boxed{\phi = \tan^{-1}(\mu_s)}$$

Note:— value of angle of friction (α) and value of angle of repose (ϕ) are same, $\boxed{\alpha = \phi = \tan^{-1}(\mu_s)}$

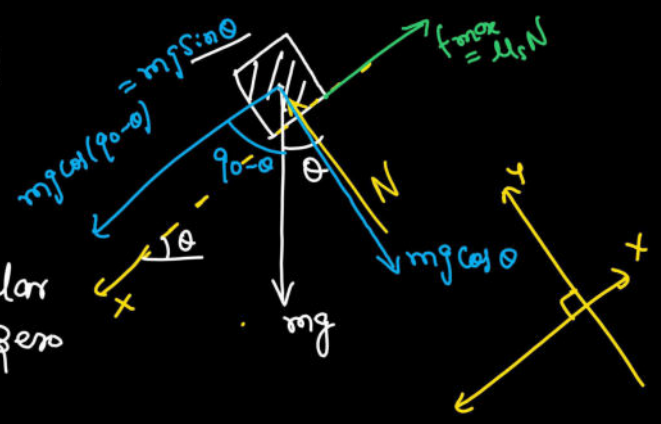
Relation between inclination angle (θ) of inclined plane and angle of repose (ϕ) :→

let θ = angle of inclination with horizontal of inclined plane.

let a block of mass (m) is in limiting eqbm in downward direction on the inclined plane.

∵ block is in limiting eqbm

∴ Total force on the block perpendicular to inclined plane is zero



$$\Sigma F_y = 0$$

$$+N - mg\cos\theta = 0$$

$$\boxed{N = mg\cos\theta}$$

$\because$ block is in limiting eqbm

$\therefore$ also net/total force parallel to inclined plane on the block is zero.

$$\Sigma F_x = 0$$

$$-mg\sin\theta + \mu_s N = 0$$

$$\mu_s N = mg\sin\theta$$

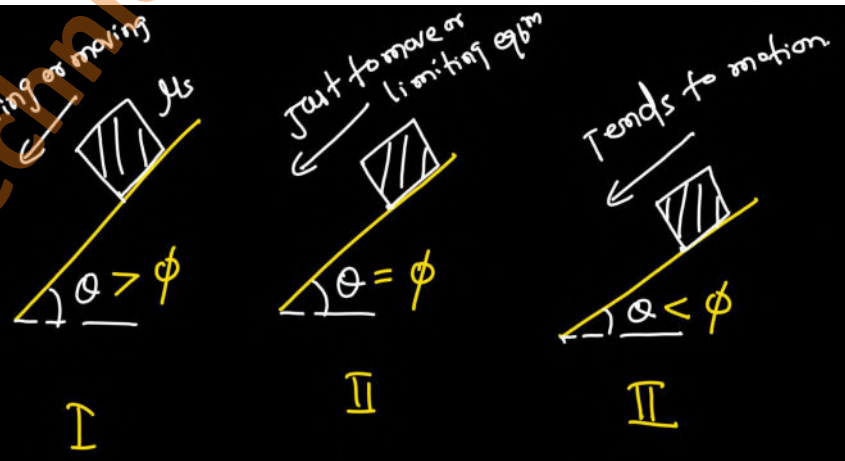
$$\mu_s \times \cancel{mg}\cos\theta = \cancel{mg}\sin\theta \qquad [\because N = mg\cos\theta]$$

$$\Rightarrow \mu_s = \frac{\sin\theta}{\cos\theta}$$

$$\Rightarrow \mu_s = \tan\theta$$

$$\Rightarrow \tan(\phi) = \tan\theta \qquad [\because \phi = \tan^{-1}(\mu_s)]$$

$$\Rightarrow \boxed{\phi = \theta}$$



Minimum and Maximum force required to maintain equilibrium on an inclined plane :→

P_{min} :→ Minimum force (P_{min}) required to maintain eqbm of the block (object) on inclined plane.

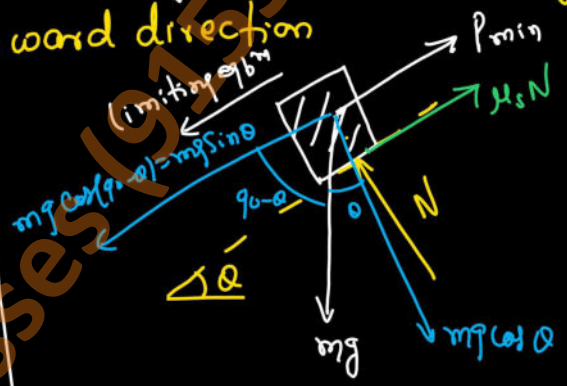
→ P_{min} will act when object is in limiting eqbm in downward direction

Net force ⊥r to plane is zero.

$\Sigma F_y = 0$

$+N - mg\cos\theta = 0$

$\boxed{N = mg\cos\theta}$



Again Net force parallel to plane is zero.

$\Sigma F_x = 0$

$P_{min} + \mu_s N - mg\sin\theta = 0$

$P_{min} = mg\sin\theta - \mu_s N$

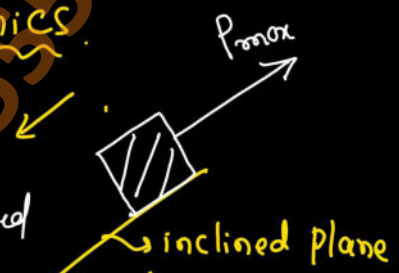
$P_{min} = mg\sin\theta - \mu_s \times mg\cos\theta$

$\boxed{P_{min} = mg[\sin\theta - \mu_s\cos\theta]}$

Engg. Mechanics

Case - II :-

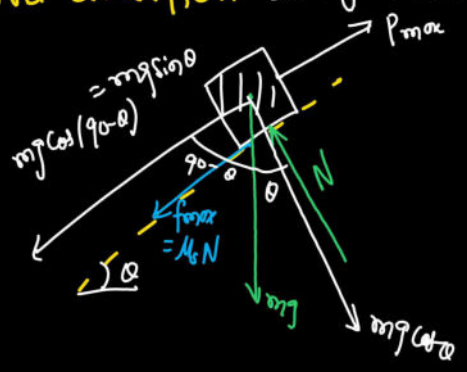
let P_{max} = Maximum force required to keep block at rest (equilibrium) on an inclined plane whose angle of inclination (θ) is greater than angle of repose (ϕ)



→ When P_{max} will acts block will come in a state of just moving upward or limiting equilibrium in upward direction along inclined plane.

∵ block/body is in eqbm on inclined plane

∴ Total/net force on the block ⊥r to inclined plane is zero.



$\because \Sigma F_y = 0$

$+N - mg\cos\theta = 0$

$$\boxed{N = mg\cos\theta} \quad \text{—— (1)}$$

Again net/total force on the block parallel to inclined plane is zero, $\Sigma F_x = 0$

$$P_{max} - mg\sin\theta - \mu_s N = 0$$

$$P_{max} = mg\sin\theta + \mu_s N$$

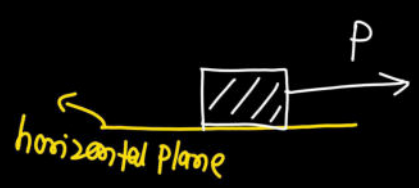
$\therefore$ putting value of normal reaction (N) from equation (1),

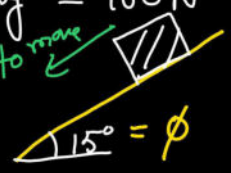
$$P_{max} = mg\sin\theta + \mu_s \times mg\cos\theta$$

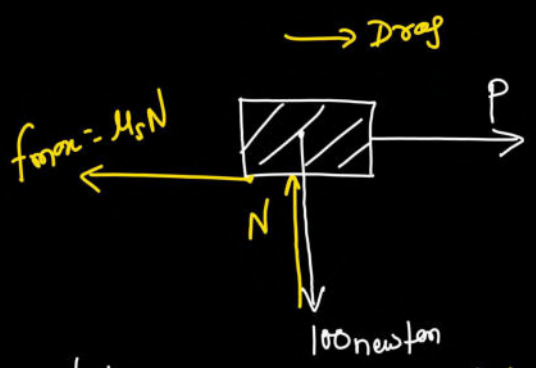
$$\boxed{P_{max} = mg[\sin\theta + \mu_s\cos\theta]}$$

Q. Find the horizontal force required to drage a body of weight 100N along a horizontal plane, if the plane when gradually raised upto 15°, the body just begins to slide.

Solution: Given weight (mg) of body = 100N







let a horizontal force (P) required to drage the body on horizontal plane

∵ total/net $\perp^r$ to horizontal plane is zero.

$$+N - 100 = 0$$

$$\boxed{N = 100 \text{ newton}}$$

To drage the body in rightward dirn.

$$P > \mu_s N$$

$$P > \mu_s \times 100$$

$$\boxed{P > 100\mu_s} \quad (1)$$

∵ angle of repose $(\phi) = 15^\circ$

$$\tan^{-1}(\mu_s) = 15^\circ$$

$$\therefore \mu_s = \tan(15^\circ)$$

$$= 0.27$$

putting this value in equation (1) we get

$$P > 100\mu_s$$

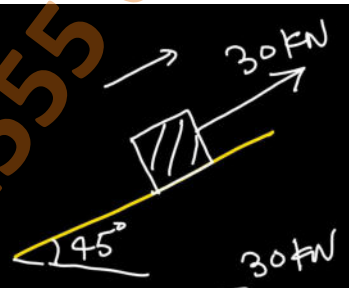
$$P > 100 \times 0.27$$

$$\boxed{P > 27 \text{ newton}}$$

Q. A weight of 35KN is pulled up on a rough inclined plane whose inclination to the horizontal is 45° by a force of 30 kN acting parallel to plane, find the co-efficient of friction.

weight of block = 35kN

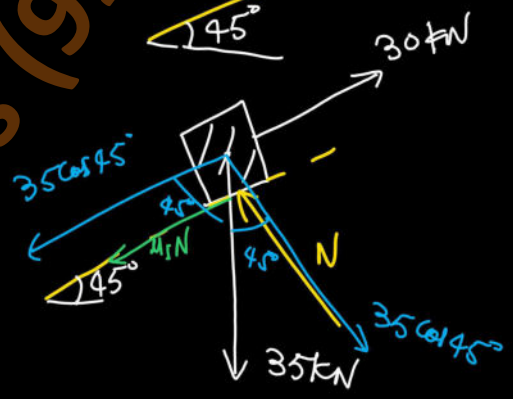
let coefficient of friction = μ_s



$\because \Sigma F_y = 0$

$\therefore N - 35\cos 45 = 0$

$\boxed{N = 35\cos 45}$ — (1)



To pulled-up,

$30 > 35\cos 45 + \mu_s N$

$30 - 35\cos 45 > \mu_s N$

$\frac{30 - 35\cos 45}{N} > \mu_s$

from eqn (1) $N = 35\cos 45$

$\Rightarrow \frac{30 - 35\cos 45}{35\cos 45} > \mu_s$

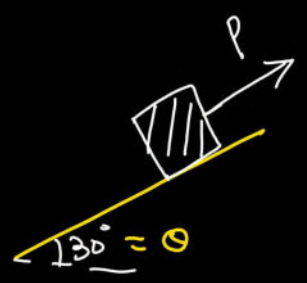
$0.212 > \mu_s$

$\therefore \boxed{\mu_s < 0.212}$ Ans//

Q. A body of mass 10 kg is on a rough inclined plane having an inclination 30° with horizontal, if co-efficient of friction between surface of contact of body and plane is 0.5, Find the least force required to pull the body up the plane.

Given mass (m) = 10 kg

$\mu_s = 0.5$



Engg. Mechanics

UNIT-04:- CENTRE OF MASS, CENTROID AND MOMENT OF INERTIA.

Centre of mass :→

→ It is a point of a body at which whole mass of the body is assume to be concentrated at this point.

→ Centre of mass of a body is take for analysis of state (rest or motion) of the body

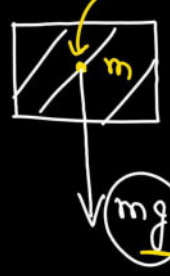
** → Centre of mass of an object/body may lies on the body and also may be lies outside the body.

** → Total/net moment about centre of mass due to its weight is always zero.

Centre of gravity (C.G) :→

where

g = Acceleration due to gravity = $9.81 \frac{m}{sec^2}$



centre of mass. = centre of gravity.

mg = weight of body = Gravitational force on the body by the Earth.

→ Centre of gravity is a point of the body at which whole weight of the body assume to be concentrated at this point of the body.

→ In general centre of mass and centre of gravity are taken as same point for a body/object

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→ Centroid of a body/object is a point of the body at which whole surface area/volume of the body is assume to be concentra-ted at this point of the body.

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Difference between centre of mass and centroid :→

CENTRE OF MASS	CENTROID
(i) Centre of mass is the point at which whole mass of the body is situated, at this point	it is point at which whole Area/length/volume is assume to be situated at this point.

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" C.G2 " " (X_2, Y_2)

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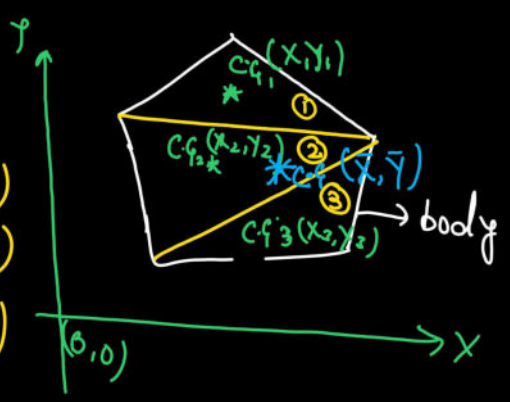

let W_1 = weight of ① part of the body.

W_2 = " " ② " " "

W_3 = " " ③ " " "

∴ Total weight of the body $\boxed{(W) = W_1 + W_2 + W_3}$

let C.G = Centre of mass/gravity of the whole body and its co-ordinates is $(\bar{X}, \bar{Y})$

∴ Now take moment of all the parts of the body about x-axis and adding these moments which will be equal to moment about x-axis of weight of whole body.

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$$\boxed{\bar{Y} = \frac{W_1 Y_1 + W_2 Y_2 + W_3 Y_3}{W_1 + W_2 + W_3}}$$

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∴ Co-ordinates of C.G of the whole body :→

$$\boxed{\bar{X} = \frac{x_1 W_1 + x_2 W_2 + x_3 W_3}{W_1 + W_2 + W_3}}$$

(x, y)

$$\boxed{\bar{Y} = \frac{y_1 W_1 + y_2 W_2 + y_3 W_3}{W_1 + W_2 + W_3}}$$

Engg. Mechanics

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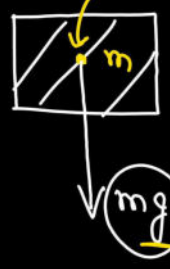

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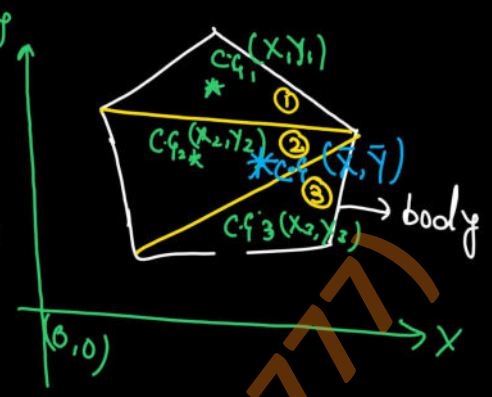
Methode of finding centre of mass of a composite body :→

Moment - Methode :→

let centre C.G₁ has co-ordinate (x_1, y_1)
" " C.G₂ " " (x_2, y_2)
" " C.G₃ " " (x_3, y_3)
" " " "



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∴ Total weight of the body $\boxed{(W) = W_1 + W_2 + W_3}$

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Engg. Mechanics

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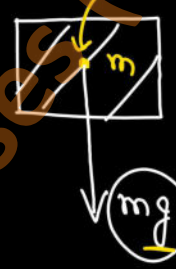

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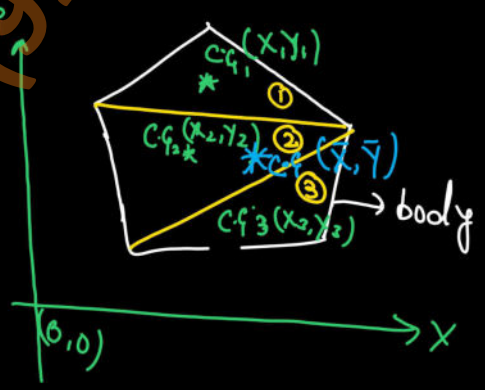

let W_1 = weight of (1) part of the body.

W_2 = " " (2) " " "

W_3 = " " (3) " " "

∴ Total weight of the body $\boxed{(W) = W_1 + W_2 + W_3}$

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$$\boxed{\bar{x} = \frac{W_1 x_1 + W_2 x_2 + W_3 x_3}{W_1 + W_1 + W_2}}$$

∴ Co-ordinates of C.G of the whole body :→

$$\boxed{\bar{x} = \frac{x_1 W_1 + x_2 W_2 + x_3 W_3}{W_1 + W_2 + W_3}} \quad (\bar{x}, \bar{y})$$

$$\boxed{\bar{y} = \frac{y_1 W_1 + y_2 W_2 + y_3 W_3}{W_1 + W_2 + W_3}}$$

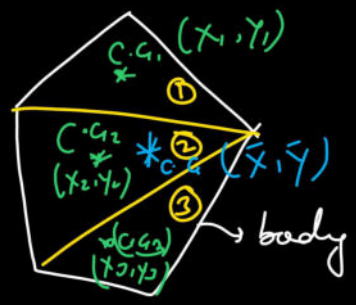
Engg. Mechanics

W_1, W_2, W_3 weight of part 1, 2, & 3

body weight $(W) = W_1 + W_2 + W_3$

$$\bar{X} = \frac{X_1 W_1 + X_2 W_2 + X_3 W_3}{W_1 + W_2 + W_3}$$

$$\bar{Y} = \frac{Y_1 W_1 + Y_2 W_2 + Y_3 W_3}{W_1 + W_2 + W_3}$$



$\because$ weight $= mg$

$= \underline{\text{density}(\rho) \times \text{Volume}(V)} \times \underline{g}$

$\therefore$ If material of the body is homogeneous. $\left[\text{density } (\rho) = \frac{\text{mass}}{\text{volume}}\right]$

i.e $\boxed{\text{density}(\rho) = \text{constant}}$

$\therefore$ weight $(W_1) = \rho \times \text{Volume}(V_1)\, g$

$W_2 = \rho \times \text{Volume}(V_2) \times g$

$W_3 = \rho \times \text{Volume}(V_3)\, g$

$\therefore$ total weight of body $(W) = W_1 + W_2 + W_3$

$= \rho \times V_1 + \rho V_2 + \rho V_3$

$= \rho g [V_1 + V_2 + V_3]$

$\because$ $W_1 = m_1 g$

$W_2 = m_2 g$

$W_3 = m_3 g$

and total weight $(W) = W_1 + W_2 + W_3 = (m_1 + m_2 + m_3) g$

where m_1, m_2, and m_3 are the masses of part 1, 2, and 3 respectively.

$$\because \bar{X} = \frac{x_1 W_1 + x_2 W_2 + x_3 W_3}{W_1 + W_2 + W_3}$$

$$= \frac{x_1 (m_1 \cancel{g}) + x_2 (m_2 \cancel{g}) + x_3 (m_3 \cancel{g})}{\cancel{g}(m_1 + m_2 + m_3)}$$

$$\boxed{\begin{aligned} \bar{X} &= \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} \\ \bar{Y} &= \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3} \end{aligned}}$$

$\Rightarrow$ Cordinate of centre of mass

$$\because \bar{X} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$$

$$= \frac{\sum_{i=1}^{\infty} m_i x_i}{M},$$ where M = total mass of body.

$$\boxed{\bar{X} = \frac{\int x\,dm}{M}}$$

Similarly, $\uparrow$ integration

$$\boxed{\bar{Y} = \frac{\int y\,dm}{M}}$$

Note: → (a) The term $\int x\,dm$ = 1st moment of mass about y-axis.

(b) $\int y\,dm$ = 1st moment of mass about x-axis.

(c) unit of 1st moment of mass is kg m.

$\therefore$ Co-ordinates of centre of mass.

$$\bar{X} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

$$= \frac{(\rho \times V_1)x_1 + (\rho \times V_2)x_2 + (\rho \times V_3)x_3}{\rho V_1 + \rho V_2 + \rho V_3}$$

$$\bar{X} = \frac{x_1V_1 + x_2V_2 + x_3V_3}{V_1 + V_2 + V_3}$$

$V \rightarrow$ volume

$$\bar{X} = \frac{\sum_{i=1}^{\infty} x_i v_i}{V} = \frac{\int x\,dv}{M}$$

$\rightarrow$ where $\int x\,dv$ = 1st moment of volume about y-axis.

= unit $\rightarrow$ meter^4.

Similarly, $\bar{Y} = \dfrac{Y_1V_1 + Y_2V_2 + Y_3V_3}{V_1 + V_2 + V_3} = \dfrac{\int y\,dv}{V}$

where $\int y\,dv \rightarrow$ 1st moment of volume about x-axis.
unit $\rightarrow$ m^4.

$$\therefore \boxed{\begin{aligned} \bar{X} &= \frac{x_1V_1 + x_2V_2 + x_3V_3}{V_1 + V_2 + V_3} \\ \bar{Y} &= \frac{Y_1V_1 + Y_2V_2 + Y_3V_3}{V_1 + V_2 + V_3} \end{aligned}}$$

$\rightarrow$ centroidal co-ordinate of 3-Dimensional volumetric object

For a planar object, made-up of homogeneous material and of constant thickness: $\rightarrow$

$$\boxed{\text{Thickness}(t) = \text{Constant}}$$

$\therefore$ volume = surface area (A) × thickness (t)

$\therefore$ $V_1 = A_1 \times t$

$V_2 = A_2 \times t$

$V_3 = A_3 \times t$

where A_1, A_2 and A_3 are area of part 1, 2 and 3.

$\therefore$ total volume (V) $= V_1 + V_2 + V_3 = (A_1 + A_2 + A_3)t$

$$\therefore \bar{x} = \frac{x_1 V_1 + x_2 V_2 + x_3 V_3}{V_1 + V_2 + V_3}$$

$$= \frac{x_1(A_1 \cancel{t}) + x_2(A_2 \times \cancel{t}) + x_3(A_3 \times \cancel{t})}{A_1 \cancel{t} + A_2 \cancel{t} + A_3 \cancel{t}}$$

$$\boxed{\bar{x} = \frac{A_1 x_1 + A_2 x_2 + A_3 x_3}{A_1 + A_2 + A_3}}$$

$$\boxed{\bar{x} = \frac{\int x\,dA}{A}}$$

where $\int x\,dA \rightarrow$ 1st moment of area 'A' about Y-axis

$\rightarrow$ unit $\rightarrow m^3$.

Similarly, $\boxed{\bar{y} = \frac{y_1 A_1 + y_2 A_2 + y_3 A_3}{A_1 + A_2 + A_3}} = \frac{\int y\,dA}{A}$

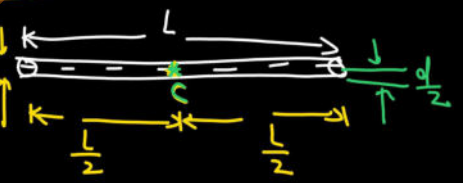
$$\bar{X} = \frac{X_1 A_1 + X_2 A_2 + X_3 A_3}{A_1 + A_2 + A_3}$$

$$\bar{Y} = \frac{Y_1 A_1 + Y_2 A_2 + Y_3 A_3}{A_1 + A_2 + A_3}$$

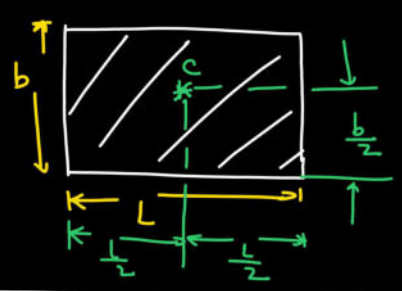
→ Centroidal Co-ordinates of Planar object/body.

Engg. Mechanics

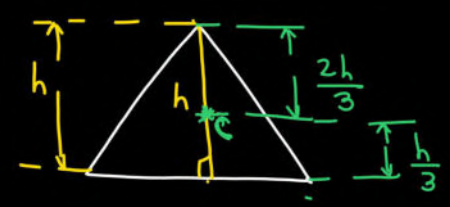
Centre of mass and Centroid of bodies made from homogeneous material: →

Geometry or Shape of body	Centroid/centre of mass
① A linear object like Rod :-	

② Rectangle

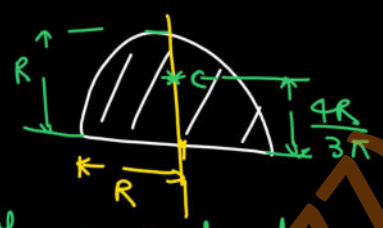


③ TRIANGLE



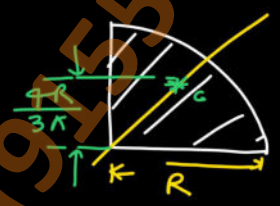
④ CIRCLE:

⑤ Semi Circle:

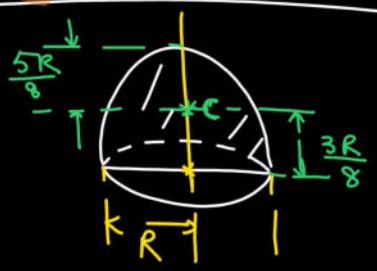


Where R = Radius of Semi circle

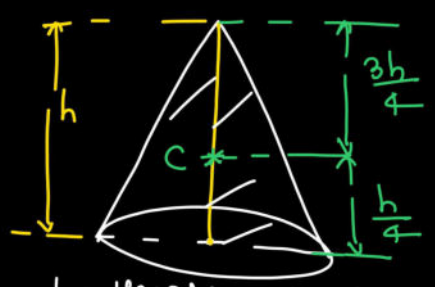
⑥ Quarter CIRCLE



⑦ Hemisphere:

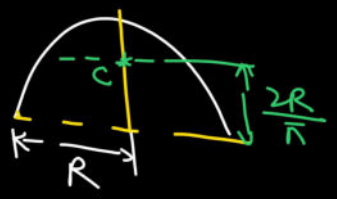


⑧ Right Circular Cone :-

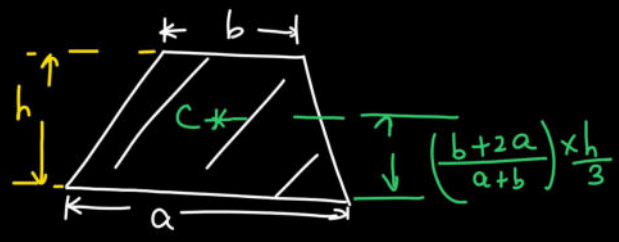


h = Height of cone.

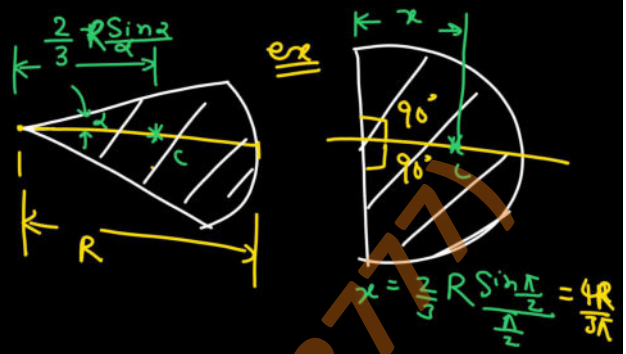
⑨ Semi Circular Arc.



(10) Trapezoidal



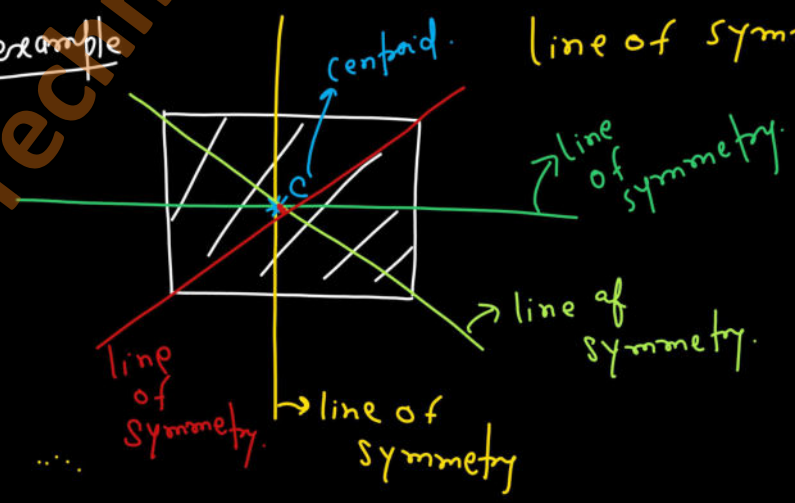
(11) Circular sector



Concept of symmetricity to Locate centroide →

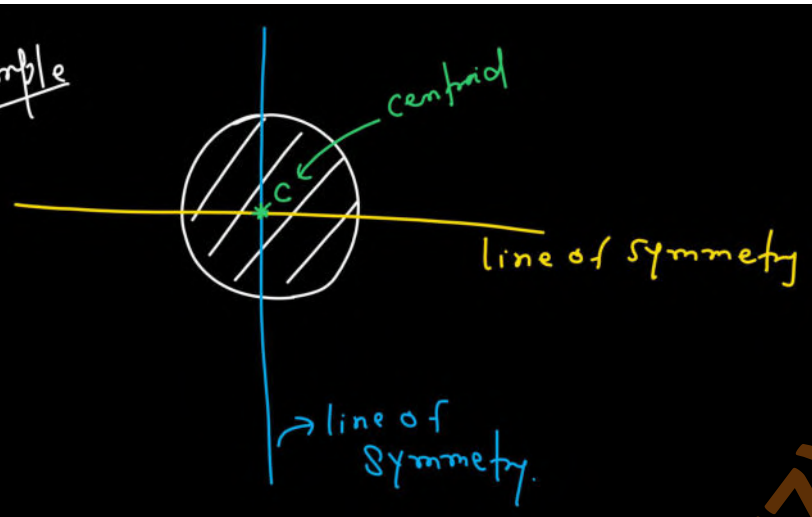
→ If any shape or geometry having at least two line of symmetry, then their centroid lies at the point of intersection of the two lines.

example

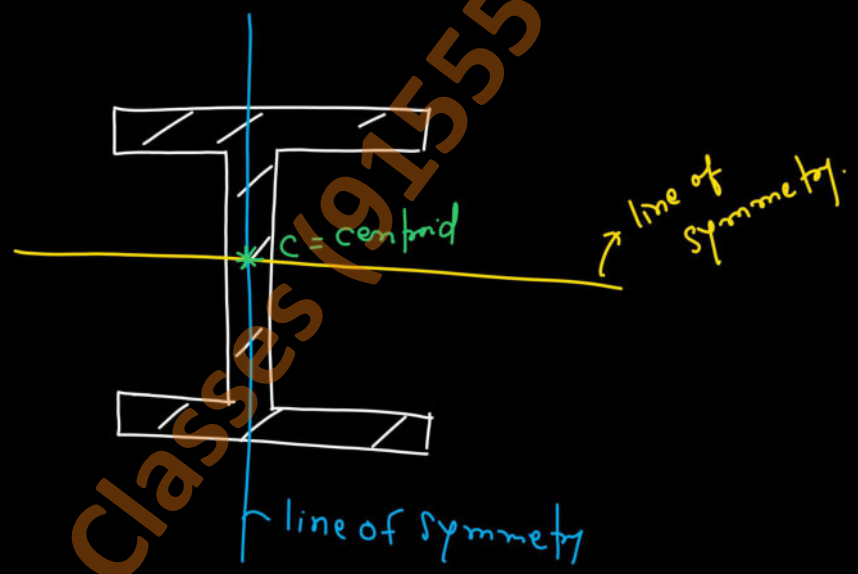


line of symmetry:—

example

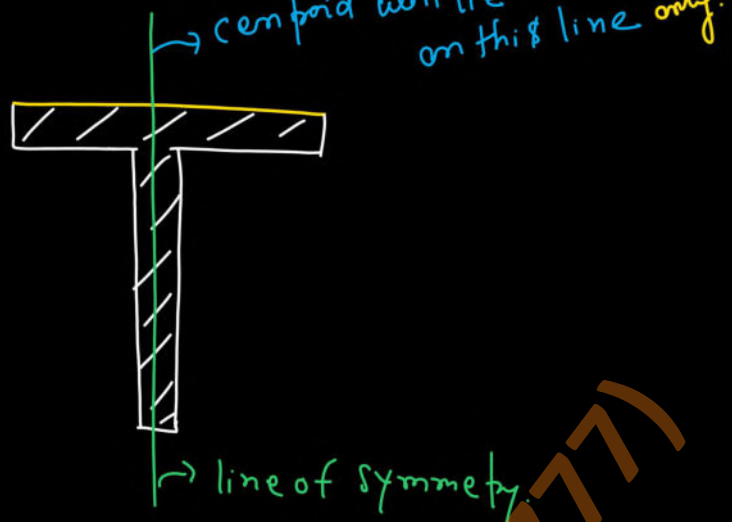


example

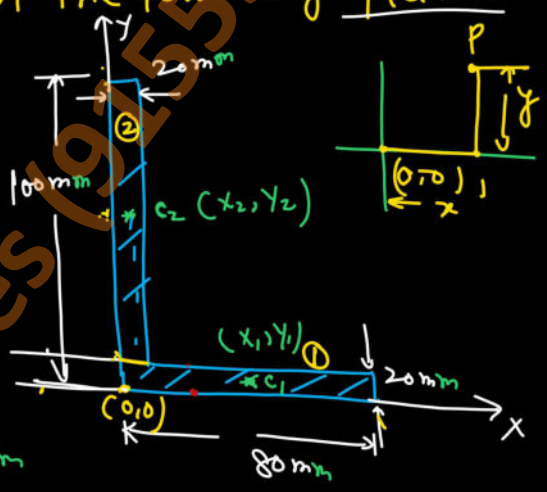


(b) If any shape or geometry of an object has only one line of symmetry, then Centroid of the body will line on that line of symmetry.

example



Q Find centroid of the following plane Area:



Sol^n :—

Area(A_1) = 80mm × 20mm
= 1600 mm^2

Area(A_2) = 80mm × 20mm
= 1600 mm^2

X_1 = 40 mm, Y_1 = 10 mm

X_2 = 10mm, Y_2 = 20 mm + 40 mm = 60 mm

let Co-ordinates of centroid of given plane geometry $(\bar{X}, \bar{Y})$

$$\therefore \bar{X} = \frac{A_1X_1 + A_2X_2}{A_1 + A_2}$$

$$= \frac{1600\,mm^2 \times 40\,mm + 1600\,mm^2 \times 10\,mm}{1600\,mm^2 + 1600\,mm^2}$$

$$\bar{x} = \frac{16 \times 40\,mm + 16 \times 10\,mm}{32}$$

$$= \frac{640\,mm + 160\,mm}{32} = 25\,mm$$

$$\bar{y} = \frac{y_1 A_1 + y_2 A_2}{A_1 + A_2}$$

$$= \frac{10 \times 1600 + 60 \times 1600}{1600 + 1600} = 35\,mm.$$

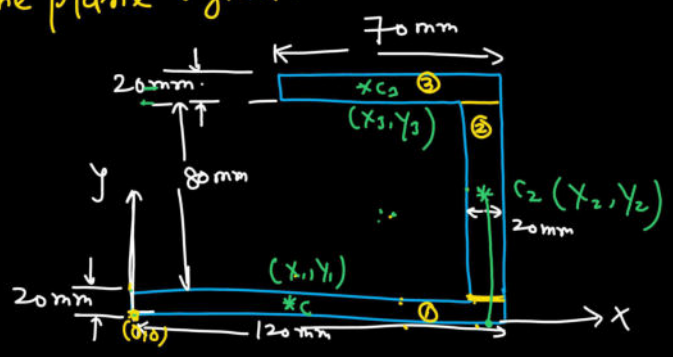
$\therefore$ Given centroid of given plane are

$(\bar{x} = 25\,mm,\ \bar{y} = 35\,mm)$

Ans//

Note** $\rightarrow$ If in a given geometry, any area which is removed from the geometry, then the value of removed area is taken with negative sign in the formula.

Q// Use the method of composite area as to calculate the centroidal coordinates of the plane regions shown in fig.



Solution

$A_1 = 120mm \times 20mm = 2400 mm^2$

$A_2 = 80mm \times 20mm = 1600 mm^2$

$A_3 = 70mm \times 20mm = 1400 mm^2$

$X_1 = 60 mm, \quad Y_1 = 10 mm$

$X_2 = 110 mm \quad Y_2 = 20mm + 40mm = 60 mm$

$X_3 = 50 + 35 = 85 mm, \quad Y_3 = 110 mm$

let $(\bar{x}, \bar{y})$ is the centroidal co-ordinates of given planar area.

$$\bar{X} = \frac{A_1X_1 + A_2X_2 + A_3X_3}{A_1 + A_2 + A_3}$$

$$= \frac{24\cancel{00} \times 60 + 16\cancel{00} \times 110 + 14\cancel{00} \times 85}{24\cancel{00} + 16\cancel{00} + 14\cancel{00}}$$

$$\boxed{\bar{X} = 81.3 \text{ mm}}$$

$$\bar{Y} = \frac{A_1Y_1 + A_2Y_2 + A_3Y_3}{A_1 + A_2 + A_3}$$

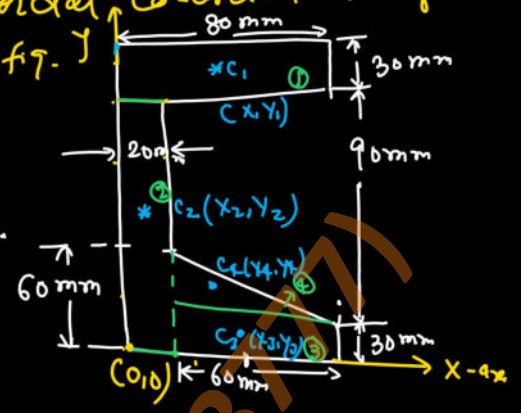
$$= \frac{24\cancel{00} \times 10 + 16\cancel{00} \times 60 + 14\cancel{00} \times 110}{24\cancel{00} + 16\cancel{00} + 14\cancel{00}}$$

$$= 50.74 \text{ mm}$$

$\therefore$ centroidal co-ordinate $(81.3, 50.3)$ Ans//

Engg. Mechanics

Q Use the method of Composite area to calculate the centroidal co-ordinates of plane region shown in fig.



Solution. $A_1 = 80\,mm \times 30\,mm = 2400\,mm^2$

$A_2 = 120\,mm \times 20\,mm = 2400\,mm^2$

$A_3 = 60\,mm \times 30\,mm = 1800\,mm^2$

$A_4 = \frac{1}{2} \times \overset{30}{\cancel{60}}\,mm \times 30\,mm = 900\,mm^2$.

$x_1 = 40\,mm$, $y_1 = 135\,mm$

$x_2 = 10\,mm$, $y_2 = 60\,mm$

$x_3 = 20\,mm + 30\,mm = 50\,mm$, $y_3 = 15\,mm$.

$x_4 = 20 + \frac{60}{3} = 40\,mm$, $y_4 = 30 + \frac{30}{3} = 40\,mm$

→ let $(\bar{x}, \bar{y})$ is the co-ordinate of centroid of given plane region.

$$\therefore \bar{x} = \frac{A_1 x_1 + A_2 x_2 + A_3 x_3 + A_4 x_4}{A_1 + A_2 + A_3 + A_4}$$

$$= \frac{2400 \times 40 + 2400 \times 10 + 1800 \times 50 + 900 \times 40}{2400 + 2400 + 1800 + 900}$$

$$\bar{x} = 32.8 \text{ mm}$$

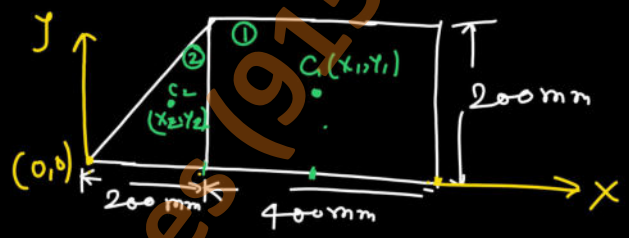
Similarly, $$\bar{y} = \frac{A_1y_1 + A_2y_2 + A_3y_3 + A_4y_4}{A_1 + A_2 + A_3 + A_4}$$

$$= \frac{24\cancel{00} \times 135 + 24\cancel{00} \times 60 + 18\cancel{00} \times 15 + 9\cancel{00} \times 40}{24\cancel{00} + 24\cancel{00} + 18\cancel{00} + 9\cancel{00}}$$

$$= 70.8 \text{ mm}$$

$\therefore$ Centroidal co-ordinate $(32.8, 70.8)$ Ans//

Q Find the centroid of the area as shown in fig.



Solution, $A_1 = 400\text{mm} \times 200\text{mm} = 80{,}000 \text{ mm}^2$

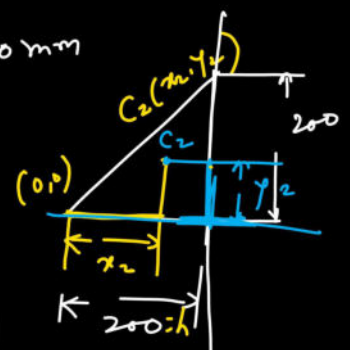
$A_2 = \frac{1}{\cancel{2}} \times 200 \times \cancel{200}^{100} = 20{,}000 \text{ mm}^2$

$x_1 = 200 + 200 = 400$ mm, $y_1 = 100$ mm

$x_2 = \frac{2 \times 200}{3} = \frac{400}{3}$ mm

$y_2 = \frac{200}{3}$ mm.

let $(\bar{x}, \bar{y})$ is the centroidal co-ordinate of given plane area.



$x_2 = \frac{2h}{3}$

$= \frac{2 \times 200}{3} = \frac{400}{3}$

$$\bar{x} = \frac{A_1x_1 + A_2x_2}{A_1 + A_2} = \frac{80000 \times 400 + 20000 \times \frac{400}{3}}{80000 + 20000}$$

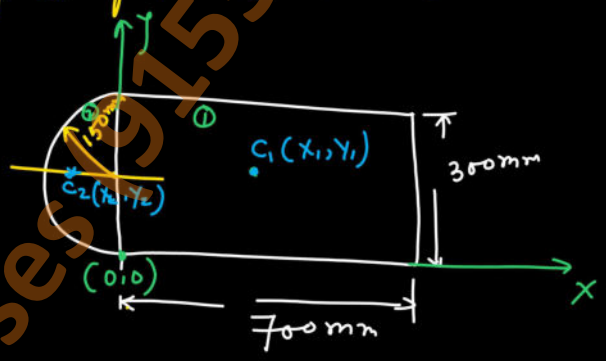
$$= 346.67 \text{ mm}$$

Similarly, $\bar{y} = \frac{A_1y_1 + A_2y_2}{A_1 + A_2} = \frac{80000 \times 100 + 20000 \times \frac{200}{3}}{80000 + 20000}$

$$\bar{y} = 93.33$$

$\therefore$ Cordinate of centroid $= (346.67, 93.33)$ Ans//

Q. Find the centroid of the section shown in fig.



$A_1 = 700 \times 300 \text{ mm}^2$

$= 210000 \text{ mm}^2$

$A_2 = \frac{\pi R^2}{2} = \frac{\pi (150)^2}{2}$

$= 35342.92 \text{ mm}^2$.

$x_1 = \frac{700\text{mm}}{2} = 350 \text{ mm}$, $y_1 = \frac{300}{2} = 150\text{mm}$

$x_2 = \frac{-4R}{3\pi} = \frac{-4 \times 150}{3\pi} = -63.66 \text{ mm}$

$y_2 = 150 \text{ mm}$

let $(\bar{x}, \bar{y})$ is the centroidal co-ordinates of given plane area.

$$\therefore \bar{x} = \frac{A_1x_1 + A_2x_2}{A_1 + A_2}$$

$$= \frac{210000 \times 350 + 35342.92 \times (-63.66)}{210000 + 35342.92}$$

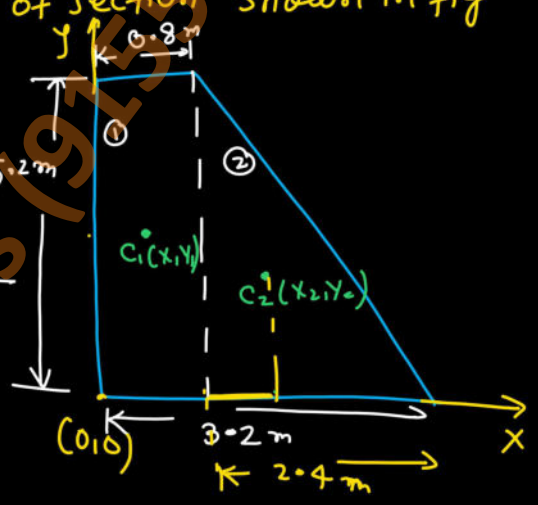
$$= 290.41 \text{ mm}$$

Similarly, $\bar{y} = \frac{A_1y_1 + A_2y_2}{A_1 + A_2} = \frac{210000 \times 150 + 35342.92 \times 150}{210000 + 35342.92}$

$$= 150$$

$\therefore$ Co-ordinate of centroid $= (290.41, 150)$ Ans//

Q// Find the Centroid of section shown in fig



Solution:

$A_1 = 5.2 \times 0.8 \text{ m}^2$

$= 4.16 \text{ m}^2$

$A_2 = \frac{1}{2} \times \overset{1.2}{\cancel{2.4}} \times 5.2 \text{ m}^2$

$= 1.2 \text{ m} \times 5.2 \text{ m}$

$= 6.24 \text{ m}^2$

$x_1 = \frac{0.8}{2} = 0.4 \text{ m}, \quad y_1 = \frac{5.2}{2} = 2.6 \text{ m}$

$x_2 = 0.8 + \frac{2.4}{3} = 0.8 + 0.8 = 1.6 \text{ m}$

$y_2 = \frac{5.2}{3} \text{ m} = 1.73 \text{ m}$

let $(\bar{x}, \bar{y})$ is the centroidal co-ordinate of given plane area.

$$\bar{x} = \frac{A_1x_1 + A_2x_2}{A_1 + A_2} = \frac{4.16 \times 0.4 + 6.24 \times 1.6}{4.16 + 6.24}$$

$$= 1.12 \text{ m}$$

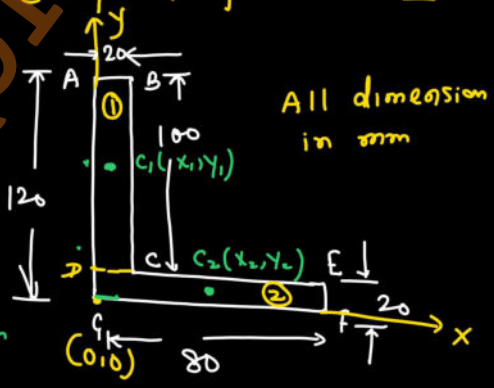
Similarly, $\bar{y} = \frac{A_1y_1 + A_2y_2}{A_1 + A_2} = \frac{4.16 \times 2.6 + 6.24 \times 1.73}{4.16 + 6.24}$

$$= 2.078$$

$\therefore$ Centroidal co-ordinate $(1.12, 2.078)$ Ans//

Engg. Mechanics

Q/ Find the location of Centroid for the L Section shown in fig.



All dimension in mm

Solution :-

$A_1 = 100 \times 20 = 2000 \text{ mm}^2$

$A_2 = 80 \times 20 = 1600 \text{ mm}^2$

$x_1 = 10 \text{ mm}, \; y_1 = 20 + 50 = 70 \text{ mm}$

$x_2 = 40 \text{ mm} \; y_2 = 10 \text{ mm}.$

let $(\bar{x}, \bar{y})$ is the co-ordinate of centre of given L Section

$$\bar{x} = \frac{A_1x_1 + A_2x_2}{A_1 + A_2} = \frac{2000 \times 10 + 1600 \times 40}{2000 + 1600}$$

$$= 23.33.$$

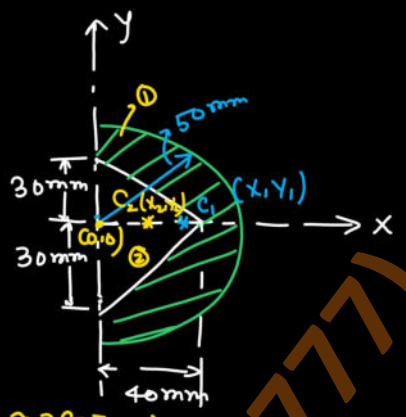
Similarly,

$$\bar{y} = \frac{A_1y_1 + A_2y_2}{A_1 + A_2} = \frac{2000 \times 70 + 1600 \times 10}{2000 + 1600}$$

$$= 43.33$$

$\therefore$ location of centroid of given L-section $= (23.33, 43.33)$ Ans//

Q Use the method of Composite area to calculate the centroidal co-ordinate of the shaded plane region shown in fig.



Solution:

let A_1 = area of whole semicircular plane region

$$= \frac{\pi r^2}{2} = \frac{\pi (50)^2}{2}$$

$$= 3926.99 \simeq 3927\ mm^2$$

A_2 = Area of triangle which is removed.

$$= \frac{1}{2} \times \cancel{60}^{30} \times 40 = -1200\ mm^2.$$

$$x_1 = \frac{4R}{3\pi} = \frac{4 \times 50}{3\pi} = 21.22\ mm$$

$$y_1 = 0$$

$$x_2 = \frac{40}{3} = 13.33$$

$$y_2 = 0$$

let $(\bar{x}, \bar{y})$ is the centroidal co-ordinate of Given shaded region.

$$\bar{x} = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2} = \frac{3927 \times 21.22 + (-1200) \times 13.33}{3927 + (-1200)}$$

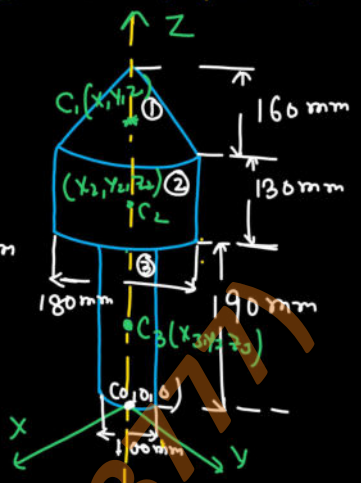
$$= 24.69\ mm$$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{3927 \times 0 + (-1200) \times 0}{3927 - 1200}$$

$$= 0$$

$\therefore$ Required centroidal co-ordinate $(24.69, 0)$ Ans//

Q. Use the method of Composite volume to determine the Centroidal co-ordinate of the volume as shown in fig.



Solution

$$V_1 = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi (90)^2 \times 160\,mm$$

$$= 1357168.026\ mm^3$$

$$V_2 = \pi r^2 h = \pi (90\,mm)^2 \times 130\,mm = 3308097.064\ mm^3$$

$$V_3 = \pi r^2 L = \pi (50\,mm)^2 \times 190\,mm = 1492256.57\ mm^3$$

$x_1 = 0,\ y_1 = 0 \quad z_1 = 190 + 130 + \frac{160}{4} = 360\ mm$

$x_2 = 0 \quad y_2 = 0,\ z_2 = 190 + \frac{130}{2} = 255\ mm$

$x_3 = 0,\ y_3 = 0,\ z_3 = \frac{190}{2} = 95\ mm.$

let $(\bar{x}, \bar{y}, \bar{z})$ is the centroidal co-ordinate of given volume.

$$\therefore \bar{x} = \frac{x_1V_1 + x_2V_2 + x_3V_3}{V_1+V_2+V_3} = \frac{0V_1 + 0\times V_2 + 0\times V_3}{V_1+V_2+V_3} = 0$$

$$\therefore \bar{y} = \frac{y_1V_1 + V_2y_2 + y_3V_3}{V_1+V_2+V_3} = \frac{0\times V_1 + 0\times V_2 + 0\times V_3}{V_1+V_2+V_3} = 0$$

$$\therefore \bar{z} = \frac{z_1V_1 + z_2V_2 + z_3V_3}{V_1+V_2+V_3} = \frac{360\times 1357168.026 + 255\times 3308097.064 + 95\times 1492256.57}{1357168.026 + 3308097.064 + 1492256.57}$$

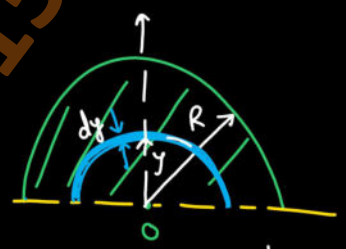
$$= 239.37\ mm.$$

$\therefore$ Required centroidal co-ordinate of given volume

$= (0, 0, 239.37)$ Ans

Centroid of Plane region Semi Circle :→

let a Semi Circle plane region of Radius R.



let a Semi Circular arc of thick 'dy' at a distance 'y' from the centre of Semi Circle.

Centroidal distance

$$\bar{y} = \frac{\int \bar{y}\, dA}{A}$$

$$= \frac{\int_0^R \frac{2y}{\cancel{\pi}} \cancel{\pi} \times y\, dy}{\frac{\pi R^2}{2}}$$

$$\bar{y} = \frac{4}{\pi R^2}\int_0^R y^2\,dy$$

$$= \frac{4}{\pi R^2}\left[\frac{y^3}{3}\right]_0^R$$

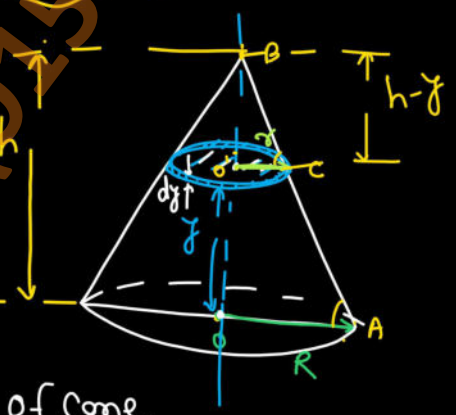
$$= \frac{4}{\pi R^2}\left[\frac{R^3}{3} - \frac{0}{3}\right]$$

$$= \frac{4R^{\cancel{3}}}{3\pi \cancel{R^2}} = \frac{4R}{3\pi}$$ Ans//

Centroid of Right Circular Cone :→

let a Right Circular Cone of height(h) and Radius 'R'

let a small disc of radius (r) and thickness 'dy' at a distance 'y' from the base of cone.



$$\bar{y} = \frac{\int_0^h y\,dv}{V}$$

$$\bar{y} = \frac{\int_0^h y \times \pi r^2\,dy}{\frac{1}{3}\pi R^2 h} \quad \text{——— (1)}$$

$\because \Delta B'C$ and ΔBOA are similar triangle

$$\frac{r}{R} = \frac{h-y}{h}$$

$$\Rightarrow r = R\left(\frac{h-y}{h}\right)$$

putting the value of 'r' in equation (1) we get

$$\bar{y} = \frac{\int_0^h y \times \cancel{\pi}\left[R\left(\frac{h-y}{h}\right)\right]^2 dy}{\frac{1}{3}\cancel{\pi} R^2 h}$$

$$= \frac{3\cancel{R^2}}{\cancel{R^2} h}\int_0^h y\left[\frac{h-y}{h}\right]^2 dy$$

$$\bar{y} = \frac{3}{h}\int_0^h y\left(1-\frac{y}{h}\right)^2 dy$$

$$\bar{y} = \frac{3}{h}\int_0^h y\left(1+\frac{y^2}{h^2}-\frac{2y}{h}\right) dy$$

$$= \frac{3}{h}\int_0^h \left(y+\frac{y^3}{h^2}-\frac{2y^2}{h}\right) dy$$

$$= \frac{3}{h}\left[\int_0^h y\,dy + \int_0^h \frac{y^3}{h^2}dy - \int_0^h \frac{2y^2}{h}dy\right]$$

$$= \frac{3}{h}\left[\left[\frac{y^2}{2}\right]_0^h + \left[\frac{y^4}{4h^2}\right]_0^h - 2\left[\frac{h^3}{3h}\right]_0^h\right]$$

$$= \frac{3}{h}\left[\frac{h^2}{2} + \frac{h\cancel{4}^{2}}{4\cancel{h}} - \frac{2h^{\cancel{3}^{2}}}{3\cancel{h}}\right]$$

$$= \frac{3}{h}\left[\frac{h^2}{2} + \frac{h^2}{4} - \frac{2h^2}{3}\right]$$

$$= \frac{3h^{\cancel{2}}}{\cancel{h}}\left[\frac{1}{2} + \frac{1}{4} - \frac{2}{3}\right]$$

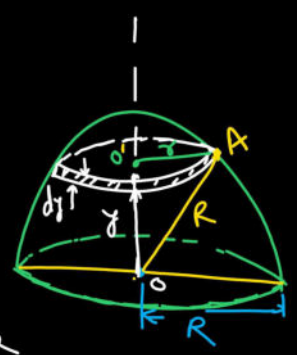
$$= 3h\left[\frac{6+3-8}{12}\right]$$

$$\bar{y} = \frac{\cancel{3}h}{\cancel{12}\,4} = \frac{h}{4}$$

$$\boxed{\bar{y} = \frac{h}{4}}$$ Ans//

Centroid of Hemisphere :→

let a hemisphere of Radius 'R' and centre 'o'



let a small disc of centre o' and radius 'r' at a distance y from the centre of hemisphere (o)

$$\bar{y} = \frac{\int_0^R y\,dv}{V}$$

$$= \frac{\int_0^R y\,\pi r^2\,dy}{\frac{1}{\cancel{2}} \times \frac{\cancel{4}^{2}}{3}\pi R^3} \quad \text{——— (1)}$$

$\because$ In $\triangle$ OO'A.

$$R^2 = r^2 + y^2$$

$$\Rightarrow \boxed{r^2 = R^2 - y^2}$$

putting the value of r^2 in equation (1) we get

$$\bar{y} = \frac{3\int_0^R \cancel{\pi} y (R^2 - y^2)\,dy}{2\cancel{\pi} R^3}$$

$$\bar{y} = \frac{3}{2R^3}\int_0^R y(R^2 - y^2)\,dy$$

$$= \frac{3}{2R^3}\int_0^R (R^2 y - y^3)\,dy$$

$$= \frac{3}{2R^3}\left[\frac{R^2 y^2}{2} - \frac{y^4}{4}\right]_0^R$$

$$= \frac{3}{2R^3}\left[\frac{R^2 \times R^2}{2} - \frac{R^4}{4}\right]$$

$$\bar{y} = \frac{3}{2 \times \cancel{R^3}} R^{\cancel{4}} \left[\frac{1}{2} - \frac{1}{4}\right]$$

$$= \frac{3R}{2} \left[\frac{2-1}{4}\right]$$

$$\bar{y} = \frac{3R}{2} \times \frac{1}{4} = \frac{3R}{8}$$

$$\boxed{\bar{y} = \frac{3R}{8}}$$ Ans//

Q// Intersection of median is the centroid of triangle. (True/False)

Ans:- True.

Q// The centre of gravity of a solid hemi-sphere of radius 'r' lies at a distance of _____ from base

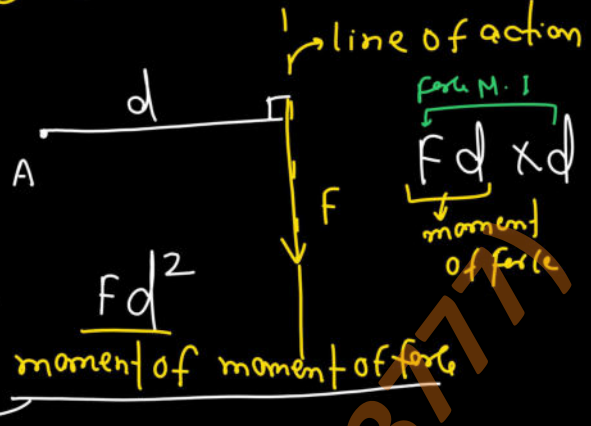
$\left(\frac{8r}{3}, \frac{3r}{4}, \frac{3r}{8}, \frac{4r}{3}\right)$

Ans:- $\frac{3r}{8}$.

Engg. Mechanics

MOMENT OF INERTIA (M.I) :→

moment



for M.I

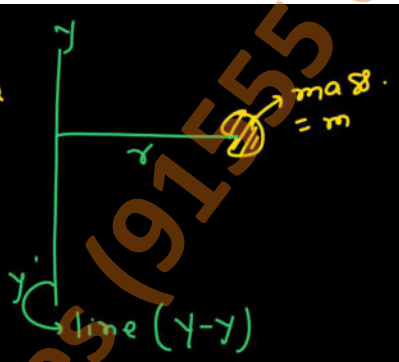
$\underbrace{Fd}_{\text{moment of force}} \times d$

For Moment of Inertia. Fd^2

moment of moment of force

⇒ Mass-moment of Inertia about line Y-Y'

$$\boxed{I_{Y-Y'} = mr^2}$$

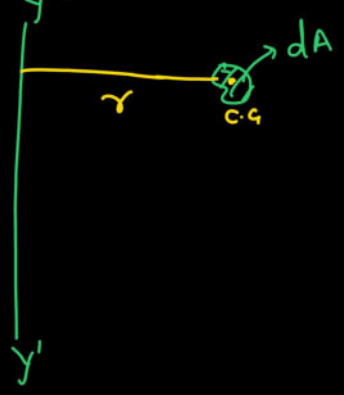


Area moment of Inertia :→

→ Second moment of area about any line is called area moment of inertia.

→ Area moment of inertia of a small area (dA) about line Y-Y' is given by

$$\boxed{I_{Y-Y'} = r^2 dA}$$

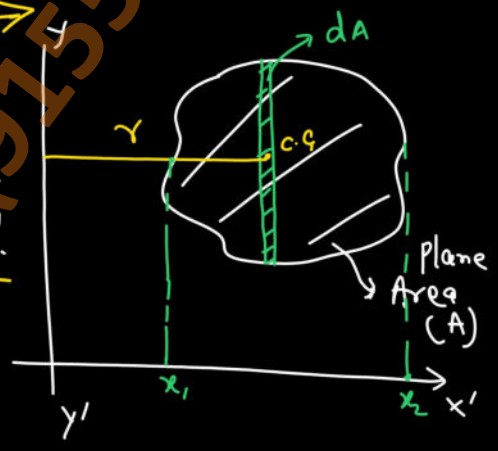


where r = distance of C.G of small area 'dA' from line y-y'.

→ Moment of inertia is a scalar physical quantity.

→ S.I unit of area moment of inertia is $meter^4$.

Integration Method to find moment of Inertia (M.I) of a plane area: →



let An area = A which moment of Inertia is required about line y-y'.

let a small element of area 'dA' which C.G is at distance 'r' from the line y-y'

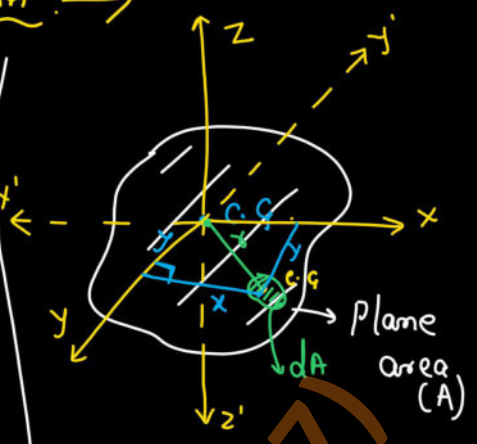
**

$$\boxed{I_{y-y'} = \int_{x_1}^{x_2} r^2 \, dA}$$

where $I_{y-y'}$ = Moment of Inertia of area 'A' about line y-y'.

Perpendicular axis theorem :→

→ This theorem states that moment of inertia of an area about line (z-z') perpendicular to plane of area and passage through C.G will be equal to sum of moment of inertia about two axis (x-x') and (y-y') which are in the plane of



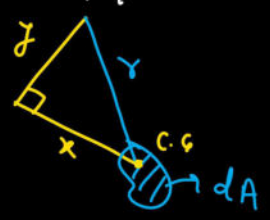
area and also passage through C.G of that area.

**

$$\boxed{I_{z-z'} = I_{x-x'} + I_{y-y'}}$$

where
$I_{z-z'}$ = Moment of Inertia about z-z'
$I_{x-x'}$ = " " " x-x'
I_{y-y} = " " " y-y'

Proof :→ let a very small area 'dA', which C.G is at a distance r, y and x from z-z', x-x' and y-y' line respectively.



$$\therefore x^2 + y^2 = r^2$$

$$\int x^2 dA + \int y^2 dA = \int r^2 dA$$

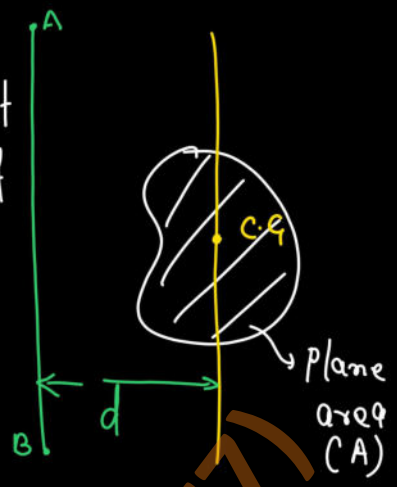
$$\boxed{I_{y-y'} + I_{x-x'} = I_{z-z'}}$$

proved//

Parallel axis theorem :→

→ This theorem states that moment of inertia of an area (A) about a line (AB) which parallel to a line passage through C.G of area (A) is given by

$$\boxed{I_{AB} = I_{C.G} + Ad^2}$$



where

I_{AB} = Moment of inertia of area (A) about line AB.

$I_{C.G}$ = Moment of inertia of Area (A) about a line which passage through C.G of area (A) and parallel line AB.

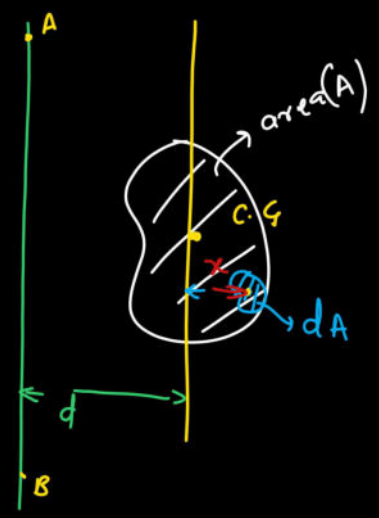
A = Area

d = Distance between two parallel lines.

Proof :→

Let a small area 'dA' which is at distance 'x' from line which passage through C.G of area (A)

Therefore moment of inertia of small area (dA) about line 'AB'



$$I_{AB} = \int (d+x)^2 \, dA$$

$$= \int \left(d^2 + x^2 + 2xd\right) dA$$

$$= \int d^2 dA + \int x^2 dA + \int 2dx dA$$

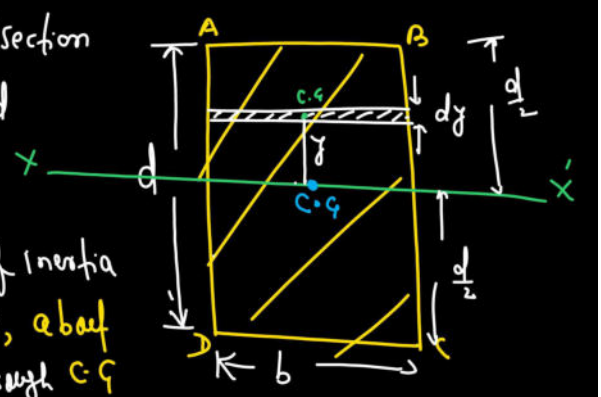
$$= d^2 \int dA + I_{C.G} + 0$$

$$= d^2 A + I_{C.G}$$

$$\boxed{I_{AB} = I_{C.G} + Ad^2}$$ proved//

Moment of Inertia of Rectangular plane area :→

let A rectangular area/section ABCD, of depth = d and width = b.



· we have find out moment of inertia of rectangular area ABCD, about a line (X-X') which passage through C.G of rectangle and cut the depth and also parallel to width.

let a small strip of width 'dy' and length 'b' whose C.G is at distance 'y' from line X-X'.

Therefore Moment of inertia of rectangular area about line X-X'.

$$I_{X-X'} = \int_{-\frac{d}{2}}^{+\frac{d}{2}} y^2 \, dA$$

$$= \int_{-\frac{d}{2}}^{\frac{d}{2}} y^2 b \, dy$$

$$I_{X-X} = b\int_{-\frac{d}{2}}^{\frac{d}{2}} y^2 \, dy$$

$$= b\left[\frac{y^3}{3}\right]_{-\frac{d}{2}}^{+\frac{d}{2}}$$

$$= \frac{b}{3}\left[\left(\frac{d}{2}\right)^3 - \left(-\frac{d}{2}\right)^3\right]$$

$$= \frac{b}{3}\left[\frac{d^3}{8} + \frac{d^3}{8}\right]$$

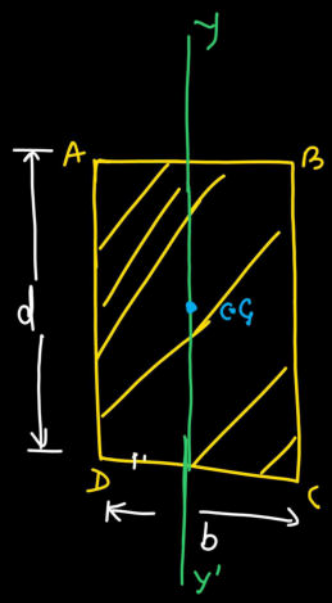
$$= \frac{b}{3} \times \frac{2d^3}{8}$$

$$\boxed{I_{X-X'} = \frac{1}{12} b d^3}$$

Similarly :→

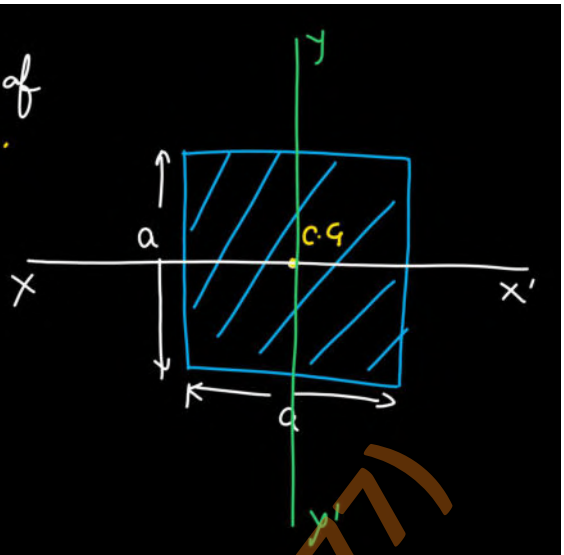
Moment of inertia of rectangle ABCD about line Y-Y' is

$$\boxed{I_{Y-Y'} = \frac{1}{12} d b^3}$$



Note:- Moment of inertia of a square of side 'a'.

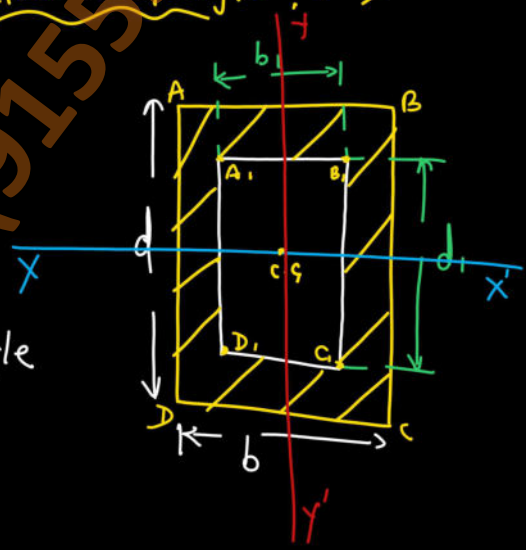
$$\boxed{I_{x-x'} = I_{y-y'} = \frac{1}{12} a^4}$$



Moment of Inertia of Hollow rectangle :→

let a rectangle ABCD of
width = b
and depth = d

let us assum, the removed rectangle $A_1B_1C_1D_1$, which
width = b_1
and depth = d_1



∴ Moment of Inertia of hollow rectangle :→

$$I_{x-x'} = \frac{1}{12} b d^3 - \frac{1}{12} b_1 d_1^3$$

$$\boxed{I_{x-x'} = \frac{1}{12}\left[b d^3 - b_1 d_1^3\right]}$$

$$\therefore \quad I_{y-y'} = \frac{1}{12} d b^3 - \frac{1}{12} d_1 b_1^3$$

$$\boxed{I_{y-y'} = \frac{1}{12}\left[d b^3 - d_1 b_1^3\right]}$$

Moment of Inertia of a Circular section/Area:—

let a Circular plane area of diameter (D) which centre is 'o'

let an axis/line which perpendicular to plane of circular area and passes through its centre, about this line moment of Inertia of circular area is given by

let a circular strip of radius (r) and thickness 'dr'

$$\therefore \quad M.I\,(I) = \int_0^{\frac{D}{2}} r^2\, dA$$

$$= \int_0^{D/2} r^2 \times 2\pi r\, dr$$

$$= 2\pi \int_0^{\frac{D}{2}} r^3\, dr$$

$$= 2\pi \int_0^{\frac{D}{2}} r^3\, dr$$

$$= 2\pi \left[\frac{r^4}{4}\right]_0^{\frac{D}{2}}$$

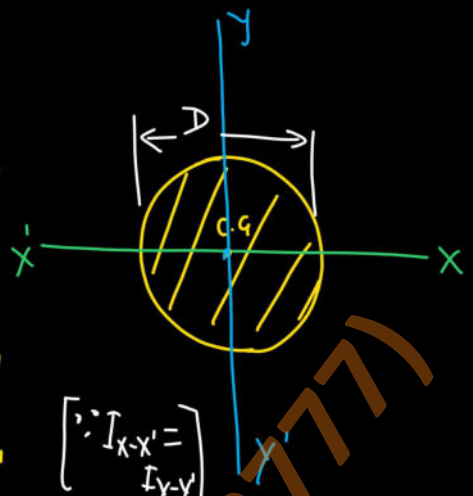
$$= 2\pi \left[\frac{\left(\frac{D}{2}\right)^4}{4} - 0\right]$$

$$= \cancel{2}\pi \times \frac{D^4}{16 \times \cancel{4}_{2}}$$

$$\boxed{I = \frac{\pi}{32} D^4}$$

Note

By perpendicular axis Theorem



$$I_{z-z} = I_{x-x'} + I_{y-y'}$$

$$\Rightarrow \frac{\pi}{32} D^4 = I_{x-x'} + I_{x-x'} \quad [\because I_{x-x'} = I_{y-y'}]$$

$$\frac{\pi}{32} D^4 = 2 I_{x-x'}$$

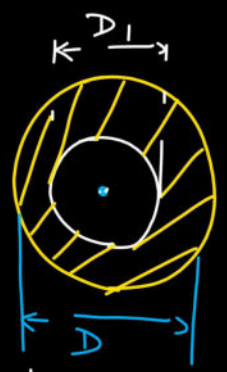
$$\boxed{I_{x-x'} = \frac{\pi}{64} D^4 = I_{y-y'}}$$

X-X' → line which in the plane of circular area and passing through centre.

Y-Y' → " " " " " "

Moment of Inertia of hollow circular section/Area:→

let a circular section of diameter 'D' out of which a small circular area of diameter D_1 is removed.



∴ M.I of remaining circular area about a line perpendicular to plane of area and passes through

Centre of area is given by.

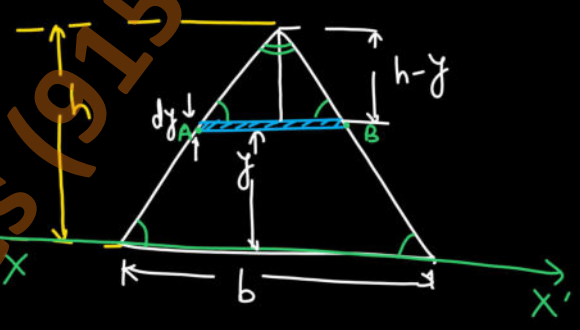
$$I = \frac{\pi}{32} D^4 - \frac{\pi}{32} D_1^4$$

$$\boxed{I = \frac{\pi}{32}\left[D^4 - D_1^4\right]}$$

Moment of Inertia of triangle :→

let a triangle of height (h) and base (b)

let a very small strip of width 'dy' at a distance y from the base line (X-X') about which moment of inertia of triangle is required.



$$I_{x-x'} = \int_0^h y^2 \, dA \quad \text{——— (1)}$$

$$\because \frac{AB}{b} = \frac{h-y}{h}$$

$$\Rightarrow \boxed{AB = b\left(\frac{h-y}{h}\right)}$$

$\therefore$ Small area $(dA) = AB \times dy = b\left(\frac{h-y}{h}\right) dy$

putting this value in equation ① we get

$$I_{x-x'} = \int_0^h y^2 b\left(\frac{h-y}{h}\right) dy$$

$$= b\int_0^h y^2\left(1-\frac{y}{h}\right) dy$$

$$= b\int_0^h \left(y^2 - \frac{y^3}{h}\right) dy$$

$$= b\left[\frac{y^3}{3} - \frac{y^4}{4h}\right]_0^h$$

$$I_{x-x'} = b\left[\frac{h^3}{3} - \frac{h^{\cancel{4}\,3}}{4\cancel{h}} - 0\right]$$

$$= bh^3\left[\frac{1}{3} - \frac{1}{4}\right]$$

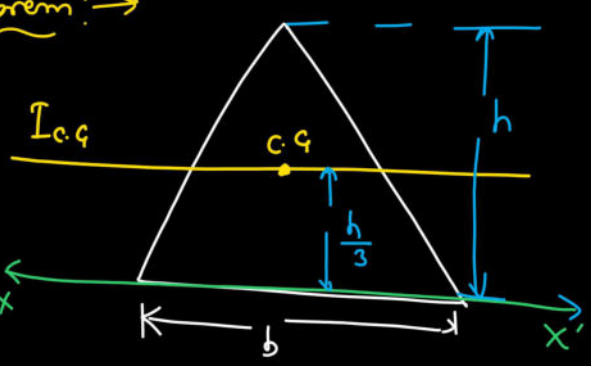
$$= bh^3\left[\frac{4-3}{12}\right]$$

$$\boxed{I_{x-x'} = \frac{1}{12} bh^3}$$

↳ moment of inertia of triangle about its base

Note:→ By parallel axis theorem:→

$$I_{x-x'} = I_{c.g} + A\left(\frac{h}{3}\right)^2$$



$$\frac{1}{12} bh^3 = I_{c.g} + \frac{1}{2} b \times h\left(\frac{h}{3}\right)^2$$

$$\Rightarrow I_{c.g} = \frac{1}{12} bh^3 - \frac{1}{2} \times \frac{bh^3}{9}$$

$$I_{c.g} = \frac{1}{12}bh^3 - \frac{1}{18}bh^3$$

$$= \frac{3bh^3 - 2bh^3}{36}$$

**

$$\boxed{I_{c.g} = \frac{1}{36}bh^3}$$

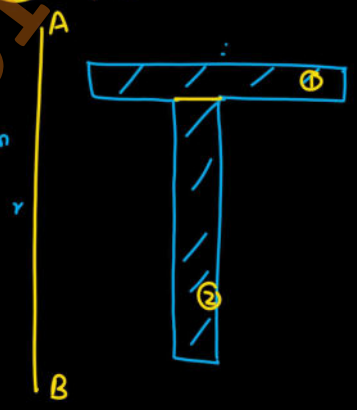
→ moment of inertia of triangle about a line passing through centre of triangle and in the plane of triangle.

Engg. Mechanics

Moment of Inertia of Composite Section/Area :→

Steps involve in finding M.I of composite section.

(a) Divide the composite section or area into small standard area.



II Find moment of inertia of each area about their own axis passing through their centroid.

III Then apply theorem of parallel axis to Calculate the moment of inertia of composite by adding the moment of inertia of each standard area about required axis.

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9PM-10PM

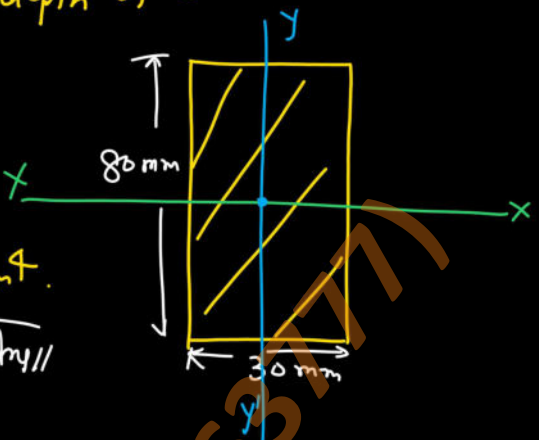
Q Find the moment of inertia of a rectangular section of width 30 mm and depth 80 mm about the axes which are passage through their centroid and parallel width and depth of section.

Solution:-

$$I_{x-x'} = \frac{1}{12}(80\,mm)^3(30\,mm)$$

$$= 1280000\ mm^4$$

Ans//



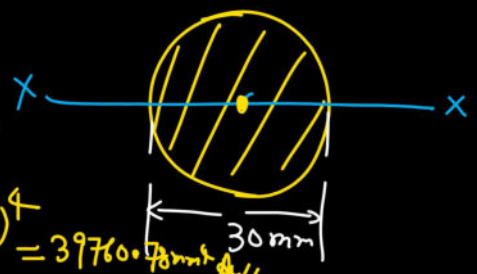
$$I_{y-y'} = \frac{1}{12}(30)^3 \times 80 = 180000\ mm^4$$ Ans//

Q Find the moment of inertia of a Circular section of diameter 30 mm about an axis passing through its centroid.

Solution:-

$$I_{x-x} = \frac{\pi}{64}(D)^4$$

$$= \frac{\pi}{64}(30\,mm)^4 = 39760.78\ mm^4$$ Ans//



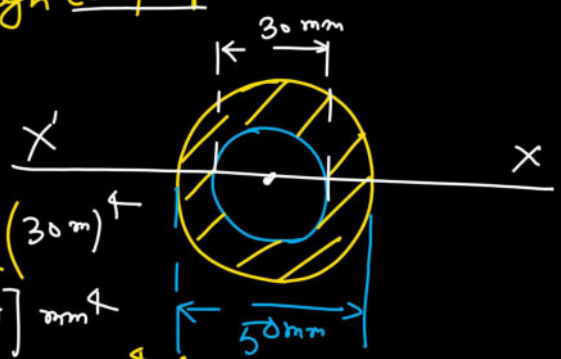
Q Find the moment of inertia of a hollow circular section of which external and internal diameter are 50 mm and 30 mm resp. about an axis which is passing through centroid circle.

Solution:-

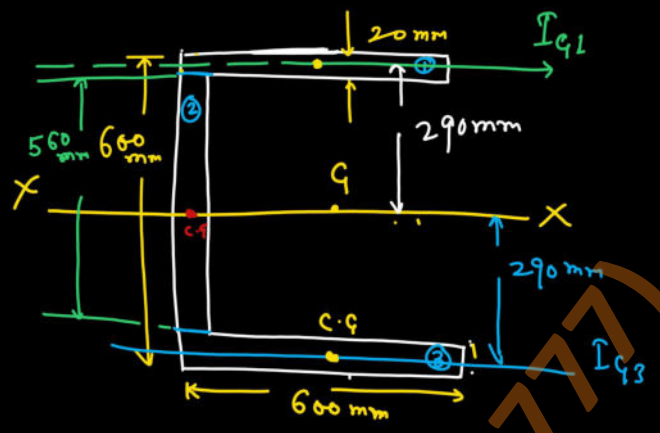
$$I_{x-x'} = \frac{\pi}{64}(50)^4 - \frac{\pi}{64}(30\,m)^4$$

$$= \frac{\pi}{64}\left[50^4 - (30)^4\right] mm^4$$

$$= 534070.75\ mm^4$$ Ans//



Q. Calculate the moment of inertia of channel section shown in figure about XX axis.



$$I_{G1} = \frac{1}{12}(20\,mm)^3 \times 600\,mm = 400000\ mm^4$$

∴ By parallel axis theorem, moment of inertia of rectangle ① about X-X -axis

$$I_{1(X-X)} = I_{G1} + A(290\,mm)^2$$

$$= 400000 + 20 \times 600 (290\,mm)^2$$

$$= 100960000\ mm^4$$

Again moment of inertia of rectangle ② about X-X -axis

$$I_{2(X-X)} = \frac{1}{12}(560)^3 \times 20\ mm^4$$

$$= 292693333.3\ mm^4$$

Again $I_{G3} = \frac{1}{12}(20)^3 \times 600\ mm^4$

$$= 400000\ mm^4$$

by parallel axis theorem:—

The moment of inertia of rectangle ③ about x-x axis

$$I_{3(x-x)} = I_{G_3} + A(290\,mm)^2$$

$$= 400000 + 20\times600(290)^2$$

$$= 100960000\ mm^4$$

∴ Therefore moment of inertia of channel section about x-x axis.

$$I_{x-x} = I_{1(x-x)} + I_{2(x-x)} + I_{3(x-x)}$$

$$= 100960000 + 292693333.33 + 100960000\ mm^4$$

$$= 2311893333.33$$ Ans//

UNIT-05

SIMPLE WEIGHT LIFTING MACHINE

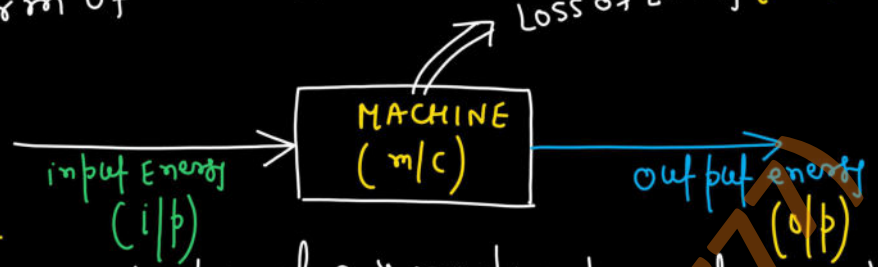
Unit-05 — 2 object x2 = 4 Marks

→ 1 Sub x 4 Marks = 4 Marks

→ 1 Sub x 6 Marks = 6 Marks

14 Marks//

MACHINE :→ Any device which takes one form of energy and convert it into other form of energy is called machines.



→ The main purpose of any machine to made easy life of human being.

Simple weight lifting machine :→

All those simple devices with the help of which heavy weight can be lifted by application of lesser force are called simple weight lifting machine.

example: pulley, wedge, lever, wheel and axel etc.

Compound weight lifting machine :→

Those machines which are formed by combination of various type of simple weight lifting machines are called compound weight lifting machine

example: Crane etc.

Load (W) :→

→ The weight which is to be lifted is called Load.

→ SI unit of load is newton (N).

Effort (P) :→ The external force applied to lift the load is called effort.

→ SI unit of effort is newton (N).

Mechanical advantage (M.A) :→

→ It is the ratio of load (W) to the effort (P).

i.e $\boxed{\text{Mechanical advantage (M.A)} = \dfrac{\text{load (W)}}{\text{effort (P)}}}$

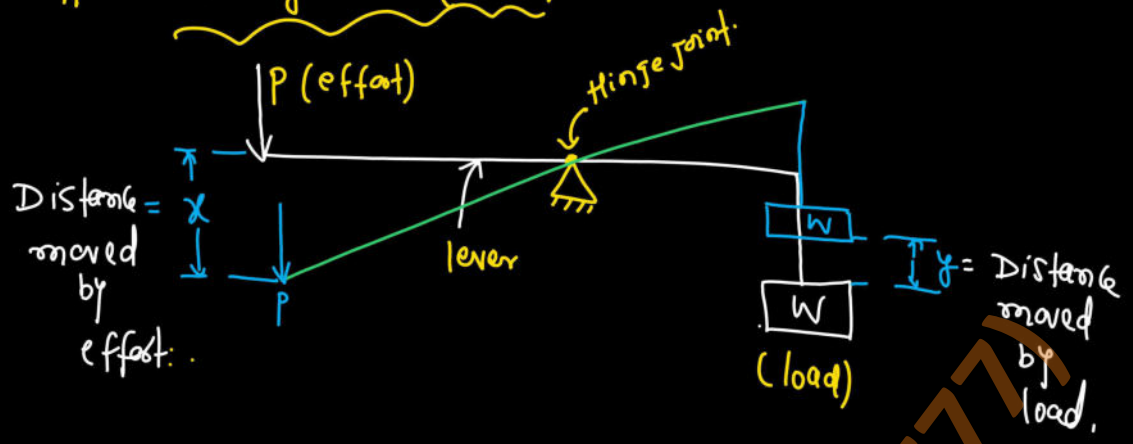
→ M.A has no unit.

→ value of Mechanical advantage is always greater than 1.

→ Mechanical advantage of compound weight lifting machine is always greater than mechanical advantage of simple weight lifting machine.

$$\boxed{(M.A)_{\text{compound M/c}} = (M.A)_1 \times (M.A)_2 \times (M.A)_3 \cdots}$$

velocity ratio (V.R) :→



→ velocity ratio is defined as the ratio of distance moved by effort (x) to the Distance moved by load (y)

i.e **velocity ratio (V.R) = $\frac{\text{Distance moved by effort } (x)}{\text{Distance moved by load } (y)}$**

input Energy (i/p) :→

→ The amount of energy given to weight lifting machine is called input energy.

input energy (i/p) = effort (P) × Distance moved by effort (x)

i/p = P × x

Output energy (o/p) :→

→ The amount energy obtained from lifting machine is called output energy.

$$\boxed{\begin{array}{l}\text{output energy} = \text{load}(W) \times \text{Distance moved by load}(y) \\ o/p = W \times y\end{array}}$$

Efficiency (η) :→ efficiency of weight lifting machine is the ratio of output energy to the input energy.

$$\boxed{\text{efficiency}(\eta) = \frac{\text{output energy}}{\text{input energy}} = \frac{W \times y}{P \times x}}$$

→ For any practical or real machine.

$$\boxed{o/p < i/p}$$

∴ efficiency (η) < 1 → for real/practical M/c.

$$\boxed{\% \eta < 100\%}$$ → " " "

Note ** A machine which efficiency is 100% (or = 1) is called Ideal or imaginary machine.

for any ideal M/c, efficiency $(\eta) = 1$

$$\frac{\text{output energy } (W \times y)}{\text{input energy } (P \times x)} = 1$$

$$\boxed{W \times y = P \times x} \rightarrow \text{for ideal m/c.}$$

Relation between mechanical advantage (M.A) velocity ratio (V.R) and efficiency (η) :→

$\because$ Mechanical advantage (M.A) $= \dfrac{\text{load } (W)}{\text{effort } (P)}$

Again $\because$ Velocity ratio (V.R) $= \dfrac{\text{Distance moved by effort } (x)}{\text{Distance moved by load } (y)}$

$\therefore$ efficiency (η) of weight lifting m/c →

$$\text{efficiency } (\eta) = \frac{W \times y}{P \times x}$$

$$= \left(\frac{W}{P}\right) \times \frac{1}{\left(\frac{x}{y}\right)} = \frac{M.A}{V.R}$$

*** $\therefore$ $\boxed{\text{efficiency } (\eta) = \dfrac{M.A}{V.R}}$

Note :- (a) :- For any Real/practical/Actual m/c

efficiency (η) < 1

$$\frac{M.A}{V.R} < 1$$

$$\boxed{M.A < V.R} \rightarrow \text{Real M/c.}$$

(b) For any ideal/imaginary machine :—

$\because$ efficiency (η) = 1

$$\frac{M.A}{V.R} = 1$$

$$\boxed{M.A = V.R} \rightarrow \text{Ideal/imaginary machine.}$$

Q. A load of 120 N is raised by means of a certain weight lifting machine, through a distance of 200 mm if the effort applied is 20 N and has moved through a distance of 1.5 m. Calculate.

(a) Mechanical advantage (M.A)

(b) Velocity ratio (V.R)

(c) efficiency. (η)

Solution :-

Given load (W) = 120 N

Distance moved by load (Y) = 200 mm.

effort (P) = 20 N

Distance moved by effort (x) = 1.5 m

$\therefore$ Mechanical advantage (M.A) $= \frac{W}{P} = \frac{\cancel{120}\,6\,\cancel{N}}{\cancel{20}\,\cancel{N}}$

$\boxed{M.A = 6}$ Ans//

(b) velocity Ratio (V.R) $= \frac{\text{Distance moved by effort (x)}}{\text{Distance moved by load (y)}}$

$= \frac{1.5\ m}{200\ mm}$

$= \frac{1.5\ \cancel{m}}{\cancel{200} \times 10^{\cancel{-3}\,-1}\ \cancel{m}}$

$= \frac{1.5 \times 10^{2}}{} = 7.5$

$\boxed{V.R = 7.5}$ Ans//

(c) Efficiency (η) $= \frac{M.A}{V.R}$

$= \frac{6}{7.5} = 0.8$

% efficiency (η) $= 0.8 \times 100 = 80\%$ Ans//

Q In a certain weight lifting machine a weight of 1 KN is lifted by an effort of 25 N, while the weight moves up by 100 mm. the point of application of effort moves by 8 m find.

(a) Mechanical advantage (M.A)
(b) velocity ratio (V.R)
(c) efficiency (η)

Solution:

Given weight/load (W) = 1 KN

effort (P) = 25 N

Distance moved by load (y) = 100 mm.

Distance moved by effort (x) = 8 m.

$\because$ Mechanical advantage (M.A) $= \frac{W}{P} = \frac{1\,KN}{25\,N} = \frac{10}{25} = 40$

$\boxed{M.A = 40}$ Ans//

Again

$\because$ velocity ratio (V.R) $= \frac{x}{y}$

$= \frac{8\,m}{100\,mm} = \frac{8\,m}{100 \times 10^{-3}\,m}$

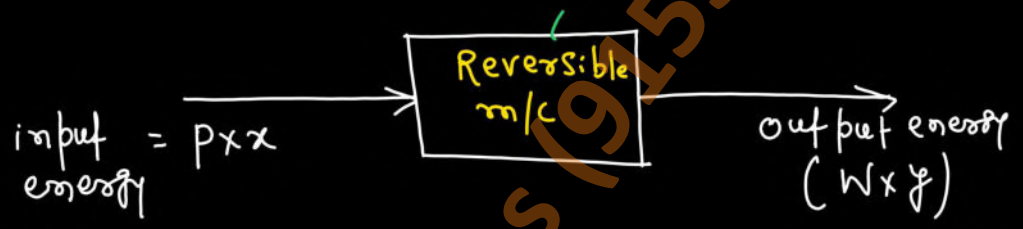
$\boxed{V.R = 80}$

(c) efficiency (η) $= \frac{M.A}{V.R} = \frac{40}{80} = 0.5$

% η = 0.5 × 100 = 50% Ans//

Reversible machine :→

→ A machine is said to be a reversible machine if it is able to do work in reverse direction if effort applied on the machine is zero.



loss of energy in M/C = input energy − output energy

$= P\times x - W\times y$

For any Reversible machine :—

output energy > loss of energy in machine.

$$W\times y > P\times x - W\times y$$

$$\Rightarrow \quad W\times y + W\times y > P\times x$$

$$\Rightarrow \quad 2W\times y > P\times x$$

$$\Rightarrow \quad \frac{W\times y}{P\times x} > \frac{1}{2}$$

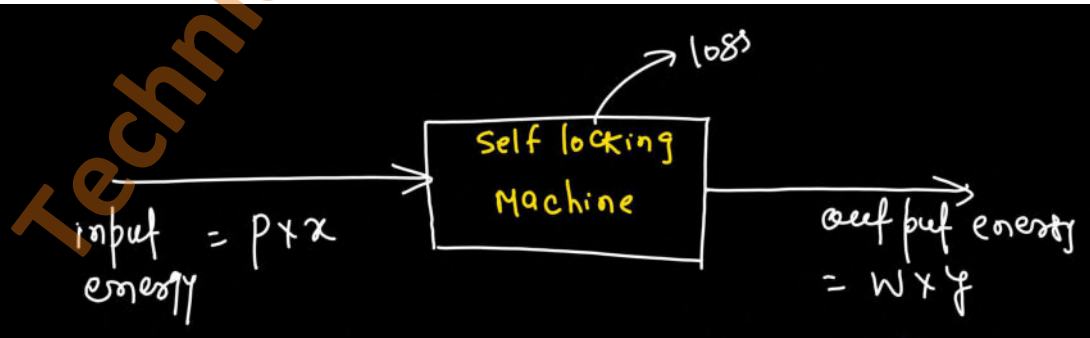
$\Rightarrow$ $\boxed{\text{efficiency}(\eta) > 0.5}$ → for Reversible machine.

For Reversible m/c.

*** $\boxed{\text{efficiency}(\eta) > 50\%}$

Irreversible machine or Self locking machine:-

→ If a machine is not able to do work in reverse direction that is which is not reversible machine is called irreversible or self locking machine.



$\therefore$ loss in machine = input − output

$= P \times x - W \times y$

for any self locking m/c: →

$$\text{output} < \text{loss}$$

$$W \times y < P \times x - W \times y$$

$$\Rightarrow W \times y + W \times y < P \times x$$

$$\Rightarrow 2 W \times y < P \times x$$

$$\Rightarrow \frac{W \times y}{P \times x} < \frac{1}{2}$$

$$\Rightarrow \boxed{\text{efficiency} < 0.5}$$ → for irreversible or self locking m/c.

or $\boxed{\% \text{ efficiency } (\eta) < 50\%}$ → " "

Q. In a weight lifting machine whose velocity ratio is 20, a weight of 1 kN can be raised by an effort of 80 N. If the effort is removed, show that the machine can work in reverse direction.

Solution: Given

velocity ratio (V.R) = 20

weight/load (W) = 1 kN

effort (P) = 80 N

$\because$ efficiency $(\eta) = \frac{M.A}{V.R} = \frac{W}{P \times V.R}$

$= \frac{1\,kN}{80 \times 20} = \frac{10^3}{80 \times 20}$

$\therefore$ efficiency $(\eta) = 0.625$

% efficiency $(\eta) = 62.5\,\% > 50\%$

Therefore machine will be reversible machine and it can work in reverse direction.

proved

Q. In a weight lifting machine an effort of 50 N is required to lift a load (W). The distance moved by load and effort are 20 mm and 500 mm respectivelly. Determine the magnitude of the load (W), if the machine is 80%.

Solution :- Given

effort (P) = 50 N

load (W) = ?

Distance moved by load (y) = 20 mm

" " " effort (x) = 500 mm

efficiency (η) = 80% = 0.8

$\because$ efficiency (η) = $\frac{M.A}{V.R}$

$$0.8 = \frac{W}{P \times \frac{x}{y}}$$

$$0.8 = \frac{W}{50N \times \frac{500\,mm}{20\,mm}}$$

$$W = 0.8 \times 50N \times 50$$

$$\boxed{W = 2000 \text{ newton}}$$ Ans//

Q// A certain weight lifting m/c has velocity ratio 30 can lift a load of 1500 N with the help of 125 N effort. Determine the machine is reversible or not.

Solution Given, V.R = 30

load (W) = 1500 N

effort (P) = 125 N

$\therefore$ efficiency $(\eta) = \frac{M.A}{V.R}$

$= \frac{W}{P \times 30}$

$= \frac{\cancel{1500N}^{\cancel{50}\,2}}{\cancel{125N}_{5} \times \cancel{30}}$

$= 0.4$

$\% \eta = 40\%$

$\therefore$ efficiency $(\eta) < 50\%$

Therefore machine is irreversible or selflocking m/c.

Q. In a weight lifting machine whose velocity ratio is 50 an effort of 100 N is required to lift a load of 4 KN, is machine reversible? If so what effort should be applied so that the machine is at the point of reversing.

Solution :-

Given

$V.R = 50$

effort(P) = 100 N

load(W) = 4KN

$\because$ efficiency$(\eta) = \frac{M.A}{V.R}$

$$\eta = \frac{W}{P \times 50}$$

$$= \frac{4KN}{100N \times 50} = \frac{4 \times 10^3}{100 \times 50} = \frac{4}{5}$$

$$= 0.8$$

% efficiency$(\eta) = 0.8 \times 100 = 80\% > 50\%$

$\therefore$ Machine will be Reversible machine //

$\rightarrow$ at the point of reversing means,

efficiency$(\eta) = 50\% = 0.5$

let P' effort is required to lift

Same load(W) = 4KN.

$\because$ efficiency$(\eta) = \frac{M.A}{V.R}$

$$0.5 = \frac{4KN}{P' \times 50}$$

$$P' = \frac{4\,KN}{0.5 \times 50}$$

$$= \frac{4 \times 10^3\ N}{25}$$

$$\boxed{P' = 160 \text{ newton}}$$ Ans//

friction in machine :→

→ friction causes loss of energy in the machine in the form of heat which results in increase of temperature of machine.

Case - I :- Friction loss in machine in terms of effort :→

let a fixed load (W) is lifted with a actual/real machine of velocity ratio (V.R) and efficiency (η)

Also, let P = effort required to lift load (W) by this actual/real machine.

then

$$\text{efficiency}(\eta) = \frac{M.A}{V.R}$$

$$\eta = \frac{W}{P \times V.R}$$

$$\Rightarrow \boxed{P = \frac{W}{\eta \times V.R}} = \text{actual effort}$$

let P' = ideal effort required to lift same load (W) by an ideal machine with same velocity ratio (V.R)

$\because$ for ideal machine, efficiency $(\eta) = 1 = \frac{M.A}{V.R}$

$$\Rightarrow 1 = \frac{W}{P' \times V.R}$$

$$\Rightarrow \boxed{P' = \frac{W}{V.R}} = \text{ideal effort.}$$

$\xrightarrow{**}$ ideal effort (P') will be less than actual effort (P) for same load.

$$\therefore \boxed{\text{friction loss in term of effort} = P - P'}$$

$$\boxed{= \frac{W}{\eta \times V.R} - \frac{W}{V.R}}$$

Case-II :- Friction loss in Machine in terms of weight :→

let P = Constant effort use to lift a load(W) by a real/actual lifting machine with velocity ratio(V.R)

let η = efficiency of actual M/c.

$$\therefore \eta = \frac{M.A}{V.R} = \frac{W}{P \times V.R}$$

$$\Rightarrow \boxed{W = \eta \times P \times V.R} = \text{Actual load.}$$

again let W' = load lifted by same effort (P) with the help of an ideal weight lifting machine with same velocity ratio (V.R)

$\therefore$ for ideal Machine,

efficiency(η) = 1

$$\frac{M.A}{V.R} = 1$$

$$\frac{W'}{P \times V.R} = 1$$

$$\boxed{W' = P \times V.R} \rightarrow \text{ideal load.}$$

** value of ideal load (W') will be greater than the value of actual load (W)

∴ Therefore, friction loss in term of load

$$\boxed{= W' - W}$$

$$\boxed{= P \times V.R - P \times V.R \times \eta}$$

Engg. Mechanics.

Q In a certain weight lifting machine an effort of 25N can lift a load of 315N. If the velocity ratio (VR) of machine is 14. Find the effort lost in friction and the friction lost in load.

Solution :- effort(P) = 25 N ✓

load (W) = 315 N ✓

V.R = 14.

let P' = ideal effort required to lift actual load (W = 315 N) by an ideal machine

∵ for an ideal machine, efficiency = 1

$$\frac{M.A}{V.R} = 1$$

$$\frac{W}{P' \times V.R} = 1$$

$$\Rightarrow P' = \frac{W}{V.R} = \frac{315\ N}{14} = \underline{22.5\ N}$$

∴ effort lost in friction = Actual effort (P) − Ideal effort (P')

= 25 N − 22.5 N

= 2.5 N Ans//

let W' = ideal load lifted by an actual effort (P = 25 N) with the help of an ideal M/c.

∵ for an ideal machine, efficiency (η) = 1

$$\frac{M.A}{V.R} = 1$$

$$\Rightarrow M.A = V.R$$

$$\frac{W'}{25} = 14$$

$W' = 14 \times 25 = 350\ N$ = Ideal load.

$\therefore$ loss of weight due to friction in machine

$= \text{Ideal load}(W') - \text{actual load}(W)$

$= 350\ N - 315\ N$

$= 35N$ Ans//

Q In a certain weight lifting machine an effort of 100N is just able to lift a load 840N. Calculate efficiency of machine and friction on both load side and effort side if the velocity ratio of machine is 10.

Solution: Given effort(P) = 100 N

load (W) = 840N

V.R = 10.

$\therefore$ we know that,

$$\text{efficiency}(\eta) = \frac{M.A}{V.R}$$

$$= \frac{W}{P \times V.R} = \frac{840N}{100N \times 10} = 0.84$$

$$\% \text{ efficiency}(\eta) = .84 \times 100 = 84\%$$ Ans//

→ friction on load side :—

let W' = ideal load lifted by actual effort ($P = 100N$) with the help of an Ideal machine.

∴ for an ideal machine,

efficiency (η) = 1

$$\frac{M.A}{V.R} = 1$$

$$\Rightarrow \quad M.A = V.R$$

$$\frac{W'}{P} = V.R$$

$$\Rightarrow \quad W' = P \times V.R = 100N \times 10 = 1000N$$

∴ friction on load side = ideal load (W') − actual load (W)

$= 1000N - 840N$

$= 160N$ Ans//

→ friction on effort side :→

let P' = ideal effort required to lift actual load ($W = 840$) with the help of an Ideal machine.

for an ideal machine,

efficiency (η) = 1

$$\frac{M.A}{V.R} = 1$$

$\Rightarrow$ M.A = V.R

$$\frac{W}{P'} = 10$$

$$\Rightarrow P' = \frac{W}{10} = \frac{840N}{10} = 84N$$

$\therefore$ friction on effort side = Actual effort(P) − Ideal effort(P')

= 100N − 84N

= 16 N Ans

Q An effort of 200 N is applied to lifting a load out of which 10% lost in friction If velocity ratio is 10. Determine

(a) load lifted

(b) efficiency of machine.

Solution :→ Given actual effort(P) = 200 N

velocity ratio (V.R) = 10

effort lost due to friction = 10%

$\therefore$ we know,

effort lost due to friction = actual effort(P) − Ideal effort(P')

= 10% of P

$$200N - P' = \frac{10}{\cancel{100}} \times \cancel{200}^{2}$$

$$200N - P' = 20N$$

$\therefore$ ideal effort $(P') = 200 - 20 = \underline{180N}$ ✓ —— (1)

let W = load lifted.

$\because$ for ideal machine, efficiency $(\eta) = 1$

$$\frac{M.A}{V.R} = 1$$

$$\Rightarrow \frac{M.A}{} = V.R$$

$$\Rightarrow \frac{W}{P'} = V.R = 10$$

$$\Rightarrow W = 10P' = 10 \times 180 \quad [\text{from eqn } (1)]$$

$$\boxed{W = 1800\ N}$$ Ans//

$\therefore$ (b) efficiency of machine $(\eta) = \frac{M.A}{V.R}$

$$= \frac{W}{P \times V.R}$$

$$= \frac{\cancel{180}^{9}\ N}{\cancel{200}N \times 10} = 0.9$$

$\therefore$ % efficiency $(\eta) = 0.9 \times 100 = \underline{90\%}$ Ans//

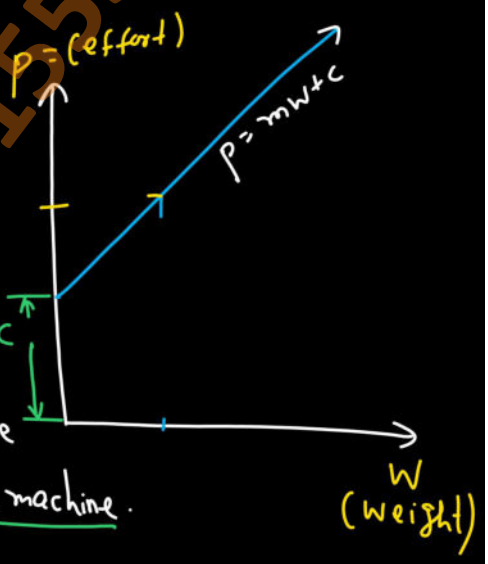
Law of machine :→

→ law of weight lifting machine is the relation between effort (P) applied and load (W) lifted.

→ law of machine is represented by a mathematical equation which is equation of law of machine and this equation is given as.

$$\boxed{P = mW + C}$$

where
P = effort applied
W = load lifted
m = constant (Coefficient of friction)
= slope of law of machine
C = represents friction of machine.



Maximum Mechanical Advantage $(M.A)_{max}$:→

∵ We know that,

Mechanical advantage (M.A) $= \frac{\text{load}(W)}{\text{effort}(P)}$

$= \frac{W}{P}$ —— (1)

By law of machine,

∴ $P = mW + C$

putting value of effort (P) in equation ① we get.

$$M.A = \frac{W}{mW + c}$$

for Maximum (M.A),

$$mW + c \approx mW$$

$$\therefore (M.A)_{max} = \frac{\cancel{W}}{m\cancel{W}}$$

$$\boxed{(M.A)_{max} = \frac{1}{m}}$$

where
m = slope of equation of law of machine
= friction co-eff.

Maximum efficiency $(\eta)_{max}$

$$\therefore \text{efficiency}(\eta) = \frac{M.A}{V.R}$$

$$\therefore \boxed{(\eta)_{max} = \frac{(M.A)_{max}}{V.R} = \frac{1}{m \times V.R}}$$

Q. What load can be lifted by an effort of 100N if the velocity ratio is 20 and efficiency of machine is 60%.
Also calculate, law of machine, maximum mechanical advantage and maximum efficiency if an effort of 200 N can lift a load of 2500 N with the help of same machine.

Solution: Given.

effort $(P_1) = 100\,N$

load $(W_1) = ?$

$V \cdot R = 20$

efficiency $= 60\% = 0.6$

Again effort $(P_2) = 200\,N$

load $(W_2) = 2500\,N$

$\because$ we know,

$$\text{efficiency} = \frac{M \cdot A}{V \cdot R}$$

$$0.6 = \frac{W_1}{P_1 \times V \cdot R}$$

$$\Rightarrow W_1 = 0.6 \times P_1 \times V \cdot R = 0.6 \times 100 \times 20$$

$$= 1200\,N$$

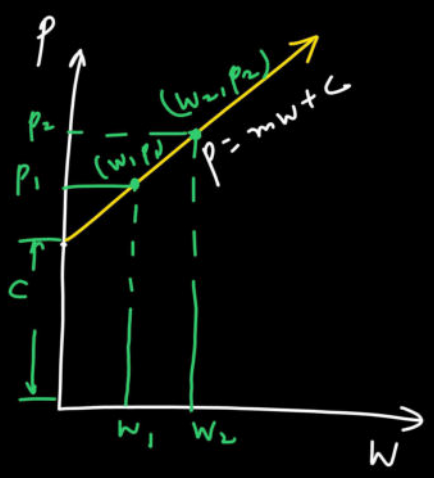
$$\Rightarrow \boxed{W_1 = 1200\,N}$$

$\therefore$ Equation of law of machine,

$$\frac{P - P_1}{P_2 - P_1} = \frac{W - W_1}{W_2 - W_1}$$

$$\frac{P - 100}{200 - 100} = \frac{W - 1200}{2500 - 1200}$$

$$\Rightarrow \frac{P - 100}{100} = \frac{W - 1200}{1300}$$



$$13P - 1300 = W - 1200$$

$$13P = W - 1200 + 1300$$

$$13P = W + 100$$

$$\Rightarrow \boxed{P = \frac{1}{13}W + \frac{100}{13}} \quad \text{— (I)}$$

→ law of machine Ans.

∵ We know,

$$P = mW + C \quad \text{— (II)}$$

∴ Comparing equation (I) and (II)

$$m = \frac{1}{13}, \quad C = \frac{100}{3}.$$

∴ Maximum (M.A) $= \frac{1}{m} = \frac{1}{1/13} = 13$ Ans//

And Maximum efficiency $(\eta)_{max} = \frac{(M.A)_{max}}{V.R} = \frac{13}{20}$

$= 0.65$

$$\boxed{\% (\eta)_{max} = 65\%}$$ Ans//

Q. What load can be lifted by an effort of 12N, If the velocity ratio is 18 and efficiency of machine at this load is 60%. If the machine has constant frictional resistance determine, the law of machine and find the effort required at

(a) No load (b) For a load of 900 N.

Solution: Given effort $(P) = 12N$

load $(W) = ?$

velocity ratio $(V.R) = 18$

efficiency $(\eta) = 60\% = 0.6$

$\because$ we know that,

$$\text{efficiency}(\eta) = \frac{M.A}{V.R}$$

$$0.6 = \frac{W}{12N \times 18}$$

$$\therefore \text{load}(W) = 12 \times 18 \times 0.6\ N = 129.6\ N \quad \text{Ans}$$

$\because$ A/R friction in machine = Constant.

let P' = Ideal effort to lift load $(W = 129.6N)$ with Ideal Machine.

$\because$ we know that for ideal machine,

efficiency $(\eta) = 1$

$$\frac{M.A}{V.R} = 1$$

$$\Rightarrow M.A = V.R$$

$$\frac{W}{P'} = V.R$$

$$\Rightarrow P' = \frac{W}{V.R} = \frac{129.6N}{18} = 7.2\ N$$

$\therefore$ friction lost in effort side $= C = P - P'$

$$= 12N - 7.2N$$

$$\boxed{C = 4.8\ N}$$

$\because$ law of machine,

$$P = mW + C$$

$$P = mW + 4.8$$

$\because$ $12N = m \times 129.6 + 4.8N$

$$m = \frac{(12 - 4.8)\cancel{N}}{129.6\cancel{N}} = 0.056$$

$\therefore$ law of machine,

$$\boxed{P = 0.056W + 4.8} \rightarrow \text{law of m/c}$$

Ans//

$\therefore$ effort required at no load,

$W = 0$

$\therefore$ from law of M/c.

$$P = 0.056 \times 0 + 4.8N$$

$$\boxed{P = 4.8N}$$ Ans//

→ (b) Effort require for 900N,

from law of machine,

$$P = 0.056W + 4.8$$

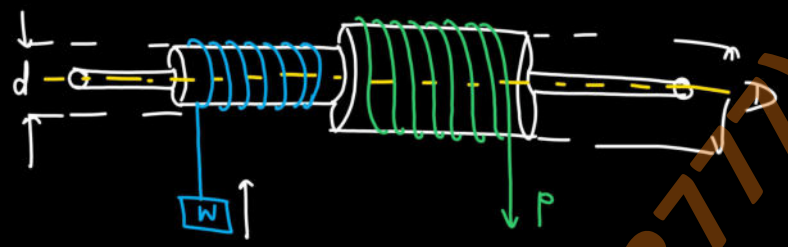
$$P = 0.056 \times 900 + 4.8$$

$$\boxed{P = 55.2\ N}$$ Ans//

Engg. Mechanics

Some Important Simple weight lifting m/c :-

** I Simple wheel and axel :-



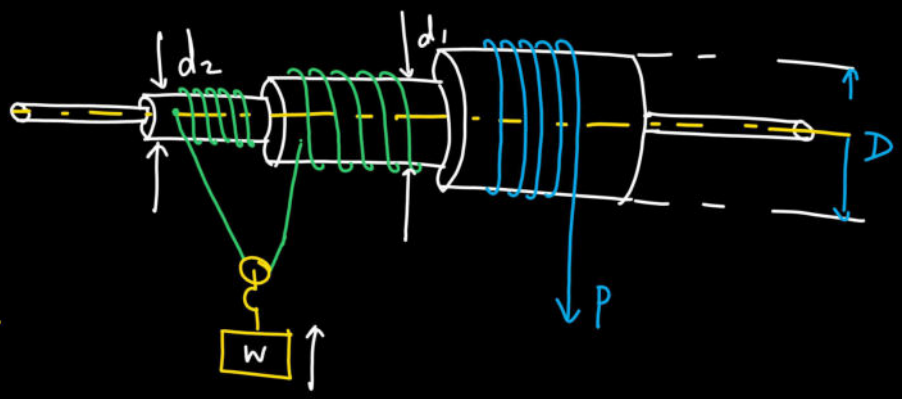
where
D = Diameter of wheel
d = diameter of Axel.
P → effort applied.
W → load which is to be lifted.

** $\therefore$ $\boxed{\text{Velocity ratio} = \frac{\pi D}{\pi d} = \frac{D}{d}}$ $= \frac{\text{Dia. of wheel}}{\text{dia. of Axel.}}$

$M.A = \frac{W}{P}$

$\therefore$ efficiency $(\eta) = \frac{M.A}{V.R}$

** 2. Differential wheel and axel :→



where D = Diameter of wheel.
d_1 = diameter of an axel
d_2 = diameter of another axel.
P = effort applied
W = load.

**

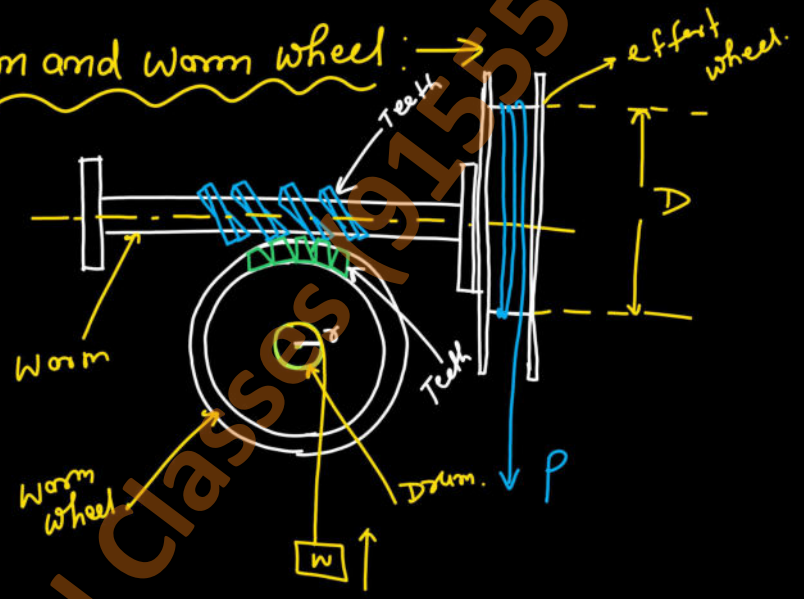
$$\therefore \boxed{(V.R) = \frac{2D}{d_1 - d_2}}$$

$$M.A = \frac{W}{P}$$

$$\text{efficiency}(\eta) = \frac{M.A}{V.R}$$

③ ***

Worm and Worm wheel :→



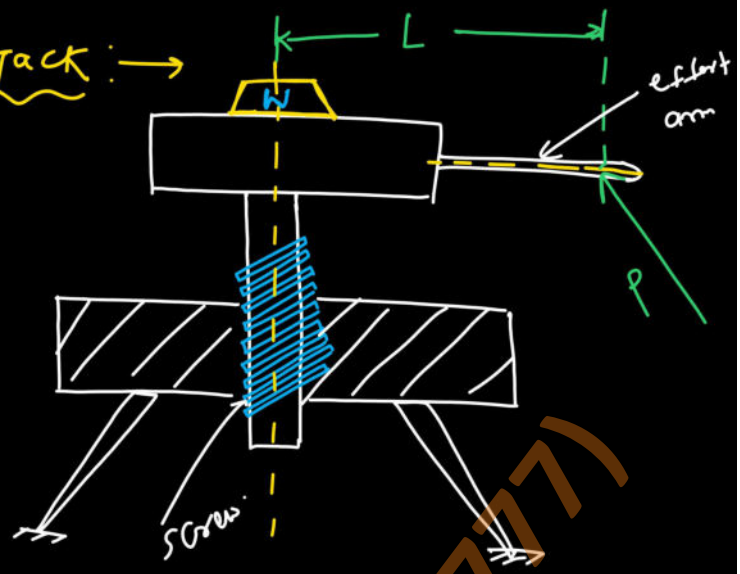
where
D = Diameter of effort wheel.
r = radius of load drum.
W = load
P = effort applied
T = total number of teeth on worm wheel

**

$$\therefore \boxed{\text{velocity ratio } (V.R) = \frac{DT}{2r}} \rightarrow \text{for single start worm wheel.}$$

$$\boxed{V.R = \frac{DT}{4R}} \rightarrow \text{for double start worm wheel.}$$

(4) Simple screw jack :→



L = length of the effort arm.

d = Pitch of screw.
= Distance of load lowered or down during one complete rotation of effort arm.

W = load lifted

P = effort applied.

∴ velocity ratio (V.R) of screw jack :—

$$V.R = \frac{2\pi l}{d}$$

∵ Mechanical advantage (M.A) $= \frac{W}{P}$

∴ $$\text{efficiency } (\eta) = \frac{M.A}{V.R}$$

⑤ velocity ratio (V.R) of first system of pulley :→

$$V.R = 2^n$$

where
n = number of pulley.

⑥ velocity ratio (V.R) of 2nd system of pulley :→

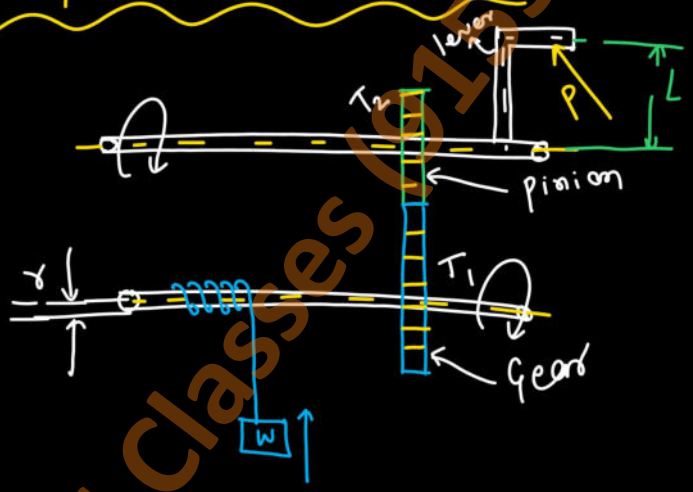
$$V.R = n$$

where
n = number of pulley in the system.

⑦ velocity ratio of 3rd system of pulley :→

$$V.R = 2^n - 1$$

⑧ Single - purchange crab winch :→



where :—

T_1 = Total number of teeth on Main Gear.

T_2 = total number of teeth on Pinion.

r = radius of load drum.

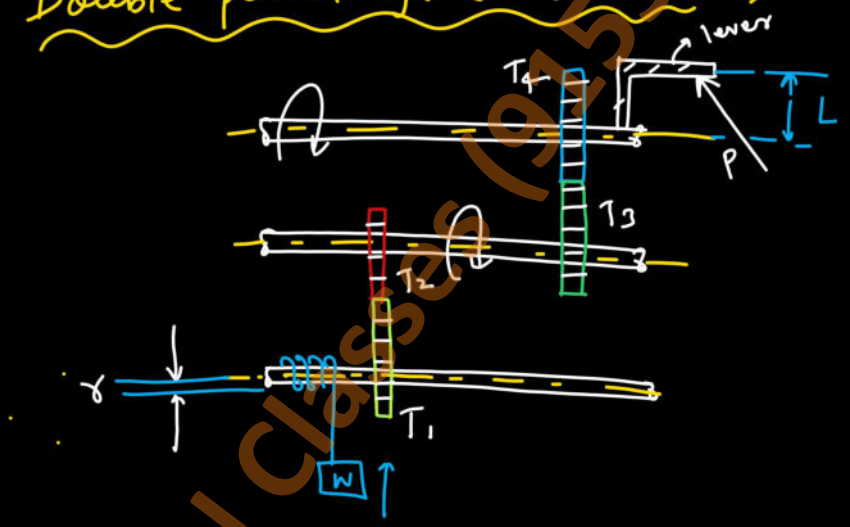
L = length of effort arm.

W = load which is to be lifted

P = Effort applied

$$\therefore \quad \boxed{\text{velocity ratio (V.R)} = \frac{2\pi L}{\frac{T_2}{T_1} \times 2\pi r} = \frac{L T_1}{T_2 r}}$$

⑨ Double purchange crab winch :→



where,

T_1, T_3 = Total number of teeth on spur Gear wheel,

T_2 and T_4 → Total number of teeth on both the pinions.

r = radius load drum.

L = length of effort arm.

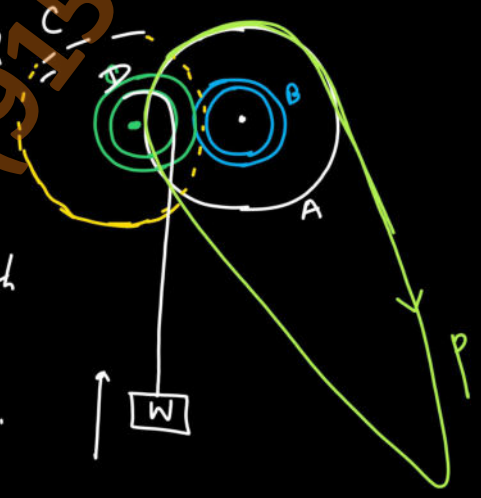
$$\boxed{V.R = \frac{2\pi L}{\frac{T_2 \times T_4}{T_1 \times T_3} \times 2\pi r}}$$

→ Double purchase crab winch.

(10) GEARED PULLEY BLOCK :-

T_1 = total number cogs/teeth on wheel 'A' or effort wheel.

T_2 = Total number of teeth on wheel 'B' which is also called pinion.



T_3 = total number of teeth on 'C' which is known as spur wheel.

T_4 = Number of cogs/teeth on wheel D.

$$\boxed{V.R = \frac{T_1 \times T_3}{T_2 \times T_4}}$$

(11) WESTON Differential pulley Block :→

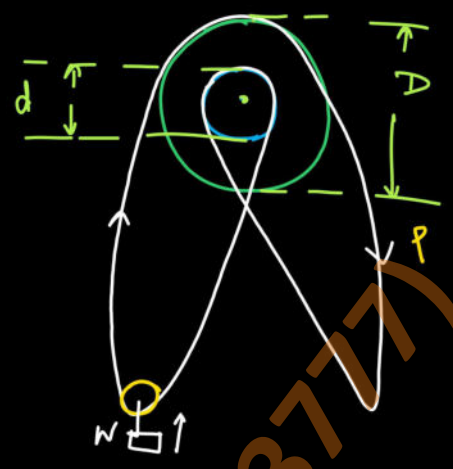
where

D = diameter of bigger pulley.

d = diameter of smaller pulley.

P = effort applied.

W = load lifted.



$$\therefore \boxed{V.R = \frac{2D}{D-d}}$$

$$\text{Mechanical adv. (M.A)} = \frac{W}{P}$$

$$\text{efficiency}(\eta) = \frac{M.A}{V.R}$$

Engg. Mechanics

Q The velocity ratio of third system of pulley with four pulleys

(8, 11, 15, 16)

1st → V.R = 2^n
2nd — " = n
3rd — $2^n - 1$

Ans:- $2^4 - 1 = 16 - 1 = 15$ Ans//

(j) Q:- efficiency of an ideal machine is equal to 50%.

(True/False)

Ans:- False.

Ideal M/C → 100
eff = 1
Rev. M/C
$\eta > 50\%$
self $\eta < 50$

Q Explain the following:-

(a) Ideal machine (2)

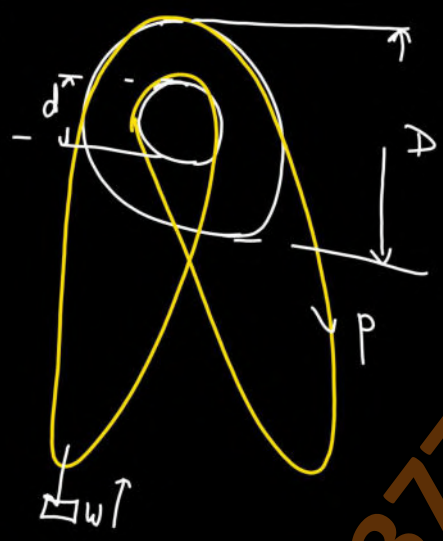
(b) friction in machine (2)

→ 4 Marks

OR

Explain weston's pulley block with neat sketch

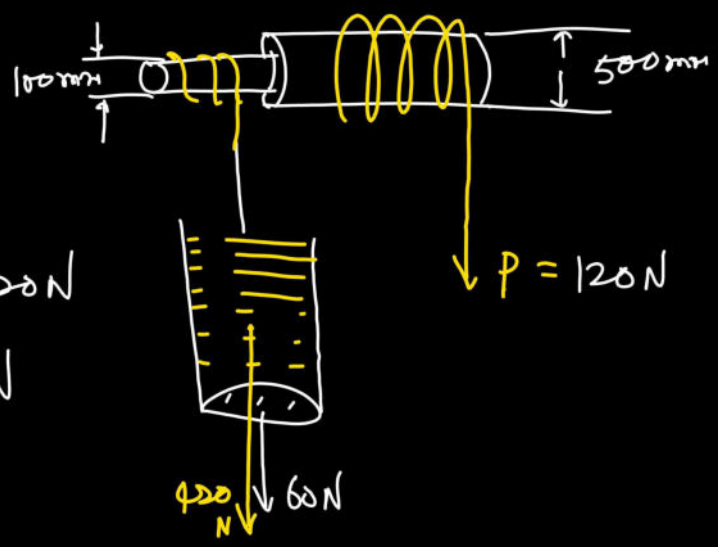
$$V.R = \frac{2D}{D-d}$$



Q A drum weight 60 N and holding 420 N water is to be raised from well by means of a wheel and axel, the axel is 100 mm diameter and wheel is 500 mm diameter, If a force of 120 N has to be applied on the wheel find (i) Mechanical advan. (ii) velocity ratio (iii) efficiency

कुआँ

Given:



Total load is to be lifted (W) = 60 N + 420 N

= 480 N

wheel dia. (D) = 500 mm

Axel dia. (d) = 100 mm

∴ (i) Mechanical advantage (M.A)

$$= \frac{\text{Load}(W)}{\text{effort}(P)} = \frac{\cancel{480}\,N}{\cancel{120}\,N} = 4 \text{ Ans}$$

(ii) velocity ratio (V.R) $= \frac{D}{d} = \frac{500}{100} = 5$ Ans

(iii) efficiency $(\eta) = \frac{M.A}{V.R}$

$$= \frac{4}{5} = 0.8$$

% efficiency $(\eta) = 0.8 \times 100 = 80\%$ Ans

OR

Write short notes on the following

(a) Differential wheel and axel. $\left(V.R = \frac{2D}{d_1 - d_2}\right)$

(b) Condition for reversiblity machine.

$$\boxed{\% \eta > 50\%}$$

Q. A machine is said to be a reversible when its efficiency is - - - -

(a) Greater than 50%.
(b) less than 50%.
(c) Both (a) and (b)
(d) None of these.

Q. The velocity ratio for third system of pulley is - - - -

(a) n
(b) n^2
(c) $2n$
(d) $2^n - 1$

1st V.R → 2^n
2nd V.R → n
3rd V.R → $2^n - 1$

Q. In a third system pulley, there are 4 pulleys. Find the effort required to lift 1800 N load, if efficiency of machine is 75%.

Solution:- Given
Third system of pulley.
number of pulley $(n) = 4$

effort (P) = ?

load (W) = 1800 N

efficiency (η) = 75% = 0.75

∵ we know that,

velocity ratio (V.R) of third system of pulley:—

$V.R = 2^n - 1 = 2^4 - 1 = 15$

∵ efficiency (η) = $\frac{M.A}{V.R}$

$0.75 = \frac{W}{P \times V.R}$

$0.75 = \frac{1800 N}{P \times 15}$

⇒ effort (P) = $\frac{1800 N}{15 \times 0.75}$ = 160 N Ans//

Q// Explain the followings

(a) law of machine

(b) Geared pulley block.

Q The VR of differential screw jack is _____ as compared to simple screw jack.

(a) Higher (b) lower (c) same (d) None of these.

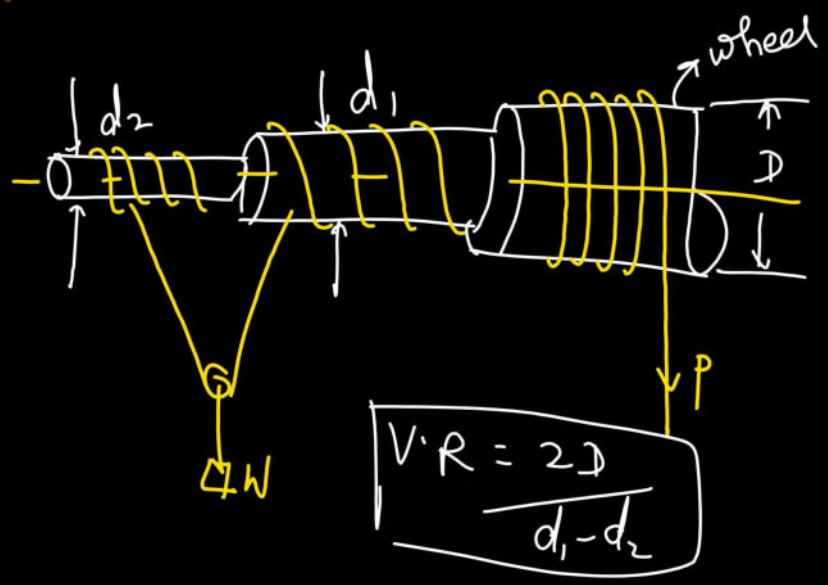
$\uparrow \eta = \frac{M.A}{V.R} \downarrow$

Q prove that efficiency of self locking machine is less than 50%. → 4 Marks.

Q state and explain reversible and non reversible machine. — 6 Marks.

OR

Draw a sketch of differential wheel and axel and write V.R.



$$V.R = \frac{2D}{d_1 - d_2}$$

Q Mechanical advantage of a compound machine comprising of two simple machines 1 and 2 is - - -

(a) $M.A = (M.A)_1 \times (M.A)_2$

(b) $M.A = (M.A)_1 + (M.A)_2$

(c) $M.A = \frac{(M.A)_1 + (M.A)_2}{2}$

(d) None.

Q For an ideal machine

(a) $MA > V.R$

(b) $MA = VR$

(c) $MA < VR$

(d) $MA = 0$

$(\eta)_{ideal} = 1 = \frac{M.A}{V.R}$

$\boxed{V.R = M.A}$ → ideal M/C

ex For Actual or real machine

$\frac{M.A}{V.R} = \eta < 1$

$\boxed{M.A < V.R}$ → Actual or real M/C.

Q. The law of certain lifting machine is $P = \frac{W}{50} + 8$, then maximum mechanical advantage is

(a) 1/50 (b) 8 (c) $\frac{1}{8}$ (d) 50.

$\because (M.A)_{max} = \frac{1}{m} = \frac{1}{1/50} = 50$ Ans//

$\because P = mW + C$ — (I)

Given $P = \frac{W}{50} + 8$ — (II)

Comparing (I) and (II)

$\therefore m = \frac{1}{50}, \quad C = 8$

Q. Ideal effort required to run a machine is - - - -

(a) more than actual effort

(b) equal to actual effort

(c) less than actual effort

(d) none of these.

(ideal effort) < (Actual effort)

(ideal load) > (Actual load)

THANK YOU ALL

GOOD LUCK.